B. E. CIVIL ENGINEERING
Choice Based Credit System (CBCS) and Outcome Based Education (OBE)
SEMESTER - VII

EARTHQUAKE ENGINEERING

Course Code	**18CV741**	CIE Marks	40
Teaching Hours/Week(L:T:P)	(3:0:0)	SEE Marks	60
Credits	03	Exam Hours	03

Course Learning Objectives: This course will enable students to learn about

1. Fundamentals of engineering seismology
2. Irregularities in building which are detrimental to its earthquake performance
3. Different methods of computation seismic lateral forces for framed and masonry structures
4. Earthquake resistant design requirements for RCC and Masonry structures
5. Relevant clauses of IS codes of practice pertinent to earthquake resistant design of structures

Module -1

Engineering Seismology: Terminologies (Focus, Focal depth, Epicenter, etc.); Causes of Earthquakes; Theory of plate tectonics; Types and characteristics faults; Classification of Earthquakes; Major past earthquakes and their consequences; Types and characteristics of seismic waves; Magnitude and intensity of earthquakes; local site effects; Earthquake ground motion characteristics: Amplitude, frequency and duration; Seismic zoning map of India; (Problems on computation of wave velocities. Location of epicenter, Magnitude of earthquake).

Module -2

Response Spectrum: Basics of structural dynamics; Free and forced vibration of SDOF system; Effect of frequency of input motion and Resonance; Numerical evaluation of response of SDOF system (Linear acceleration method), Earthquake Response spectrum: Definition, construction, Characteristics and application; Elastic design spectrum.

Module -3

Seismic Performance of Buildings and Over View of IS-1893 (Part-1): Types of damages to building observed during past earthquakes; Plan irregularities; mass irregularity; stiffness irregularity; Concept of soft and weak storey; Torsional irregularity and its consequences; configuration problems; continuous load path; Architectural aspects of earthquake resistant buildings; Lateral load resistant systems. Seismic design philosophy; Structural modeling; Code based seismic design methods.

Module -4

Determination of Design Lateral Forces: Equivalent lateral force procedure and dynamic analysis procedure. Step by step procedures for seismic analysis of RC buildings using Equivalent static lateral force method and response spectrum methods (maximum of 4 storeys and without infill walls).

Module -5

Earthquake Resistant Analysis and Design of RC Buildings: Typical failures of RC frame structures, Ductility in Reinforced Concrete, Design of Ductile Reinforced Concrete Beams, Seismic Design of Ductile Reinforced Concrete column, Concept of weak beam-strong column, Detailing of Beam-Column Joints to enhance ductility, Detailing as per IS-13920. Retrofitting of RC buildings

Earthquake Resistant Design of Masonry Buildings: Performance of Unreinforced, Reinforced, Infill Masonry Walls, Box Action, Lintel and sill Bands, elastic properties of structural masonry, lateral load analysis, Recommendations for Improving performance of Masonry Buildings during earthquakes; Retrofitting of Masonry buildings.

Course outcomes: After studying this course, students will be able to:

1. Acquire basic knowledge of engineering seismology.
2. Develop response spectra for a given earthquake time history and its implementation to estimate response of a given structure.
3. Understanding of causes and types of damages to civil engineering structures during different earthquake scenarios.
4. Analyze multi-storied structures modeled as shear frames and determine lateral force distribution due to earthquake input motion using IS-1893 procedures.
5. Comprehend planning and design requirements of earthquake resistant features of RCC and Masonry

structures thorough exposure to different IS-codes of practices.

Question paper pattern:

- The question paper will have ten full questions carrying equal marks.
- Each full question will be for 20 marks.
- There will be two full questions (with a maximum of four sub- questions) from each module.
- Each full question will have sub- question covering all the topics under a module.
- The students will have to answer five full questions, selecting one full question from each module.

Textbooks:

1. Pankaj Agarwal and Manish Shrikande, "Earthquake resistant design of structures", PHI India.
2. S.K. Duggal, "Earthquake Resistant Design of Structures", Oxford University Press
3. Anil K. Chopra, "Dynamics of Structures: Theory and Applications to Earthquake Engineering", Pearson Education, Inc.
4. T. K. Datta, "Seismic Analysis of Structures", John Wiley & Sons (Asia) Ltd.

Reference Books:

1. David Dowrick, "Earthquake resistant design and risk reduction", John Wiley and Sons Ltd.
2. C. V. R. Murty, Rupen Goswami, A. R. Vijayanarayanan & Vipul V. Mehta, "Some Concepts in Earthquake Behaviour of Buildings", Published by Gujarat State Disaster Management Authority, Government of Gujarat.
3. IS-13920 – 2016, Ductile Detailing of Reinforced Concrete Structures Subjected to Seismic Forces, BIS, New Delhi.
4. IS-1893 – 2016, Indian Standard Criteria for Earthquake Resistant Design of Structures, Part-1, BIS, New Delhi.
5. IS- 4326 – 2013, Earthquake Resistant Design and Construction of Buildings, BIS, New Delhi.
6. IS-13828 – 1993, Indian Standard Guidelines for Improving Earthquake Resistance of Low Strength Masonry Buildings, BIS, New Delhi.
7. IS-3935 – 1993, Repair and Seismic Strengthening of Buildings-Guidelines, BIS, New Delhi.

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INTRODUCTION TO SEISMOLOGY

Topics covered

- Definition of Earthquake and Terminologies
- Importance of Earthquake Engineering
- Continental Drift
- Elastic Rebound Theory
- Plate Tectonic Theory
- Tectonic Plate Boundaries
- Faults
- Surface and Body Waves
- Seismic Waves and their propagation
- Causes for Earthquake
- Types of Earthquakes
- Magnitude and Intensity
- Strong Motion Characteristics of earthquake
- Typical earthquake ground motion
- Characteristics of ground motion necessary for design
- Seismic Instrumentation
- Seismic Zoning map of India – Background, Basis and Zone factor
- Design Basis Earthquake and Maximum Credible Earthquake
- Concept of different levels of design
- Importance of Local site effect
- Microzonation
- Base Isolation
- Liquefaction

1.1 EARTHQUAKE

is the disturbance that happens at some depth below the ground level which causes vibrations at the ground surface. These vibrations happen in all the directions and are totally uncertain. The location, time, duration, magnitude and frequency of earthquake are totally unknown. Also, these vibrations are momentary, happening for a short while. It should be noted that earthquakes are totally unpredictable. Earthquake is the shaking or trembling caused by the sudden release of energy below the ground. It is usually associated with faulting or breaking of rocks. Continuing adjustment of position results in aftershocks. Fig. 1 explains some terminologies in the field of earthquake engineering.

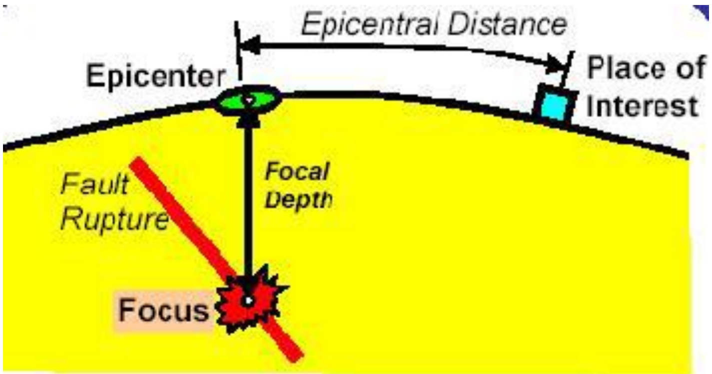
1.1.1 Focus or Hypocenter: It is the location from where earthquake originates. The point within Earth where faulting begins is the focus, or hypocenter. It may be a point, line or a plane. It will be deep below the earth surface.

1.1.2 Epicenter: It is the projection of focus on the surface of earth. It is a point which is closest to point of release of energy. The point directly above the focus on the surface is the epicenter.

1.1.3 Focal Depth: Distance between focus and epicenter is the focal depth. The closer the focal depth, more damaging is the earthquake.

1.1.4 Epicentral Distance: Distance between point of interest and epicenter is called Epicentral Disatnce.

Fig. 1 : Terminologies in Earthquake Geotechnical Engineering



Some vital statistics about Great Japan earthquake of March 2011

Magnitude : 9.0

Intensity : > X

Date : Friday, the 11th March 2011

Time : 11.30 am in Japan (8.00 am IST)

Focal Depth : 24.4 km

Region : Near east coast of Honshu Island, Japan, 130 km east of Sendai, 178 kn east of Yamagata, 178 km east north east of Fukushima, 373 km North east of Tokyo

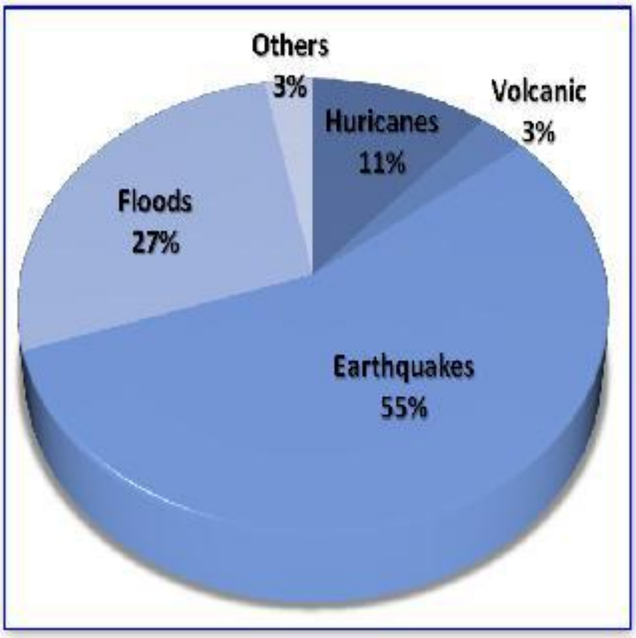
Death Toll : More than 25000

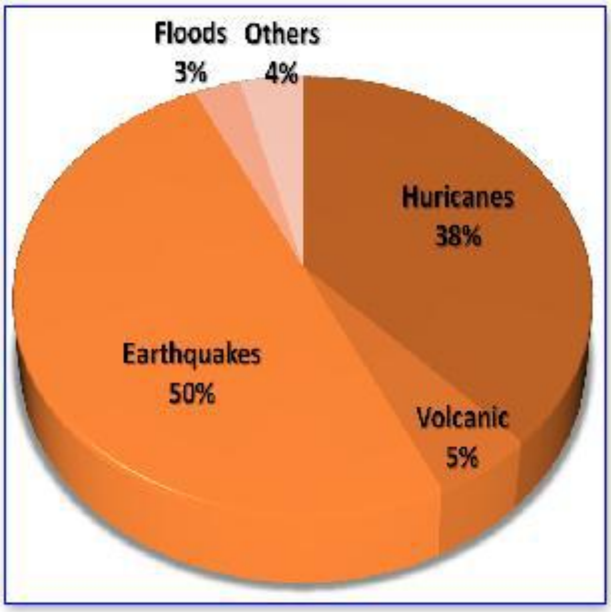
Evacuated : About 5 Lakh People
Infrastructure : Entire towns were wiped off the map, Houses, cars, ships, buildings were washed away, roads buckled, highway collapsed, power line tangled, railway track damaged (1.2 Lakh houses damaged, 15000 houses completely destroyed)
Insured loss : USD 35 Billion (175000 Crore Rupees)
Overall loss : USD 350 Billion (1750000 Crore Rupees)
Honshu Island moved by 2.4 m
Duration of shaking 3 to 5 minutes
Number of after shocks > 400 and some with magnitudes of 7.2
Change in length of day caused by redistribution of earth mass : 1.8 microsecond shorter
It can therefore be inferred that we are still in the process of understanding the nature. Nature this time was furious on Japan and resulted in a very large earthquake, fourth biggest.
Top five earthquakes ever recorded on earth were

1. Mw9.5 Chile, 5th May 1960, 1600 Killed, 20 Lakh Homeless
2. Mw 9.2 Prince William Sound, Alaska, 27th March 1964, 128 Killed, Tsunami
3. Mw 9.1 Sumatra, 26th Dec 2004, 2.2 Lakh Killed
4. Mw 9.0 Kamchatka Peninsula, Russia, 4th Nov 1952
5. Mw 9.0 Tohoku earthquake, Japan, 11th March 2011

1986 Chernobyl disaster ranked 7, which is the highest in terms of severity in Nuclear Radiation. Fukushima Power Plant disaster was also ranked more than 6 for Nuclear radiation indicating that the severity of radiation in Japan was close to the worst.
Table 2 presents some of the popular earthquakes that were eye openers to researchers, policy makers and general public. Each of these earthquakes had some special features that helped in enhancing the knowledge. Always, it is possible to learn from failures and the below detailed earthquakes caused many failures

EARTHQUAKE ENGINEERING is a relatively new branch of engineering that manages the problems caused during earthquake. The main objective is to reduce the damaging effects of earthquake, possibly warn against expected earthquake and provide suitable mitigation measures. Earthquake Engineering is interdisciplinary and requires the association of structural engineers, hydraulic engineers, geotechnical engineers, mechanical engineers, geologists, administrators, managers, bureaucrats, politicians, medical doctors, environmentalists etc. Fig. 3 explains the interdisciplinary link of earthquake engineering andthe topics covered by each group. Further, Fig. 4 indicates that earthquake is the most devastating of all the natural disasters both in terms of loss of life and loss to built environment.





Percentage Loss of Life Percentage Damage to Built Environment

Fig. 4 : Loss of Life and Damage to Built Environment during different Natural disasters in percentage

ELASTIC REBOUND THEORY

Stresses continue to build in rocks at great depths below the ground at high temperature and pressure. The following processes are expected to happen.

– Rocks bends until the strength of the rock is exceeded

– Rupture occurs and the rocks quickly rebound to an undeformed shape

– Energy is released in waves that radiate outward from the fault

This release of energy is expected to cause earthquake. When earthquake happens, slip takes place resulting in changes in positions. Fig. 5 explains the concept of Elastic Rebound Theory.

PLATE TECTONIC THEORY

About 95% of all earthquakes occur along the plate boundaries. Most of these result from convergent margin activity. Remaining 5% occur in interiors of plates and on spreading ridgecenters. More than 150,000 quakes strong enough to be felt are recorded each year. Surface of earth is made of 12 major plates – constantly drifting over semi molten mass of mantel. Plates collide causing the stresses to develop. When the Strain energy due to deformation is greater than that of resilience, then, the energy is released. The released energy is in the form of waves. Gravity and density differences, external processes such as hydrologic cycle, erosion and internal processes such as mantle convection create dynamic process in earth. Fig. 6 indicates the internal process due to mantle convection very similar to pressure build up in a pressure cooker. **1.5 CONTINENTAL DRIFT**

Alfred Wegener (1912) indicated that large supercontinent (Pangaea) existed and then split into pieces. The existing fossils and glacial deposits are the evidence. Wegener was not able to provide mechanism for his theory. Major mechanism was later found. The details are as follows.

• There is a noticeable jigsaw fit between many continents. For example, between the East Coast of South America and the West Coast of Africa, there exists matching fit. It suggests that the continents were once assembled together.

• A number of identical fossils have been found distributed across the southern continents. Fossils of the Mesosauras dating back 280 million years ago are found in South America and Africa. Plant fossils, such as Glossopteris (a tree) have been found in South America, Africa, India and Australia.

• A number of continents show evidence of matching geological sequences with rocks of similar age, type, formation and structure occurring in different countries.

• A number of climatic anomalies are discovered which suggest that continents must once have been in a different position and therefore have experienced a different climate. Coal which only forms under wet / warm conditions has been found beneath the Antarctica ice cap and there is evidence of glaciation in Brazil.

Hence, the continents were once joined. Therefore, they must have moved apart over time. Wegener proposed a mechanism for continental drift, the pushing of continents by gravitational forces that derived from the sun and the moon (similar to tides). Fig. 8 presents jigsaw matching and similar fossil presence in different continents.

SEISMIC WAVES

When the energy is released at the hypocenter or focus, it translates in to waves and travels through the body of earth. A similarity can be brought with a pebble thrown in to still water in a lake developing rings of waves in all directions. These waves attenuate after some distance and time due to material damping of earth.

There are two types of waves, namely,

– Body waves : Primary and Secondary waves

– Surface waves : Raleigh and Love waves

Body waves travel through the body of earth. P or primary waves are the fastest waves that travel through solids, liquids, or gases. These are compressional waves and material movement is in the same direction as wave movement. S or secondary or shear waves are slower than P waves. They travel through solids only. The material movement is perpendicular to wave movement.

Surface Waves are produced at the earth surface. They travel just below or along the ground's surface. They are slower than body waves and cause rolling and side-to-side movements, especially causing damage to buildings. Different waves travel at different speeds and they arrive at different instants of time at a place.

Body Waves Surface Waves

INSTRUMENTS FOR SEISMIC MEASUREMENTS

Typical seismic instrument consists of a three directional sensor, a GPS, a memory unit and a battery backup. Fig. 13 provides an idea of working of seismic instruments. Two types of seismic instruments are available. Present day instruments are very compact and are both accurate and precise. Broadly they are divided in to two categories.

1. Seismographs are generally used by seismologists or geologists. They are very sensitive and can trace the farthest earthquakes (several thousands of km away from the instrument station). However, they are not very accurate in representing the shaking at the instrument station. A seismogram is a graph of wave amplitude Vs. Time. In old seismographs, a pen drew the recording on a piece of paper. In new seismographs, the signal is recorded digitally.

2. Strong motion Accelerographs are generally used by civil engineers. They are triggered when the level of acceleration due to shaking at a place crosses the threshold acceleration. They are not sensitive, but can record very accurately the shaking parameters at a site. The graph of ground motion versus time is called accelerogram**EARTHQUAKE MAGNITUDE**

When rocks shift suddenly along a fault, they generate waves. These waves shake the ground, producing earthquakes. Seismographs record the wave amplitudes, which are used to calculate the earthquake magnitude and the energy released by the rupture.

The intensity of shaking is one way to assess the size of an earthquake. A value is assigned based on damage reports and personal interviews of people who experienced the quake. The intensity depends on location. In general, the closer the observer to the earthquake, the higher will be the intensity. Intensity values assist in seismic hazard and historical earthquake analysis.

In 1935, Charles Richter developed a method to compare the sizes of California earthquakes based on waves recorded by seismographs. In his method, a single magnitude is assigned based on the maximum wave amplitudes. Modern seismologists have modified his method and now analyze a large section of the waves recorded on a seismograph to calculate a seismic moment. The seismic moment is then converted to moment magnitude, which is the standard size reported by the U.S. Geological Survey.

The magnitude of an earthquake suggests the power or Strength of an earthquake. It is a nonzero positive number in logarithmic scale. It is a measure of strain energy released at hypocenter. It is determined by seismographs. The magnitude is independent of place.

Richter Scale is the most popular scale, according to which magnitude M is equal to,

$M = \log_{10} A$

Energy released at focus E is given by,

$\log_{10} E = 11.4 + 1.5\,M$

Each increase in M by a quantity one, increases the energy by 32 times. The atom bombs that were dropped on Hiroshima and Nagasaki cities of Japan in 1945 during the second world war had the magnitude of 5.0.

1.10.1 Moment Magnitude Scale (MMS or Mw) is most used presently. This magnitude is based on seismic moment of the earthquake. $M_w = \mu A_o D$ is a better measure for bigger earthquakes. It is equal to the rigidity of the earth (μ) multiplied by the average amount of slip on the fault (D) and the size of the area that slipped (A_o). Richter scale suffers from saturation for bigger earthquakes and hence not accurate for assessing bigger earthquakes of magnitudes 7 and higher.

1.11 EARTHQUAKE INTENSITY:

It is more a qualitative measure and not a quantitative measure of earthquake that estimates the damage. In India, modified Mercalli's scale is popular. It is a measure of damaging effect of earthquake at a site. It depends on

- Local soil conditions,
- Type and Quality of structures,

• Epicentral distance.

• Focal Depth

• Knowledge of earthquake engineering in the region.

• Performance of earthquake resistant structure.

CAUSES FOR EARTHQUAKE

The cause for an earthquake is mostly natural. But, there can be man made reasons for earthquake. The following is the list of causes for earthquakes.

• Tectonic earthquake

• Volcanic earthquake

• Rock fall or collapse of cavity

• Microseism

• Explosion (Controlled blast)

• Reservoir induced earthquake

• Mining induced earthquake

• Cultural Noise (Industry, Traffic etc.)

It should be noted that earthquake by itself may not create problems. But, it develops such a force that the man made system may not sustain under this force unless proper care is taken. The following are some characteristics of earthquake.

• An earthquake does not cause death or injury by itself.

• People are hurt by falling plaster and collapsing walls or falling of heavy objects.

• Collapsing buildings and vibrations can cause short circuits and electric fires.

• Lighted gas or stoves may also cause fires.

• All this leads to panic and confusion.

• With some precautions it is possible to avoid such confusion.

1.13 CLASSIFICATION OF EARTHQUAKES

Earthquakes can be broadly classified in to following subclasses.

1. Based on Focal Depth
2. Based on magnitude
3. Based on origin
4. Based on location
5. Based on Epicentral distance

1.13.1 Based on Focal Depth

• Shallow Focus earthquakes (<70 km)

• Intermediate focus earthquakes (70 to 300 km)

• Deep focus earthquakes (> 300 km)

1.13.2 Based on magnitude

- Micro earthquakes (M < 3)
- Intermediate earthquakes (M 3 to 5)
- Moderate earthquakes (M 5 to 6)
- Strong earthquakes (M 6 to 7)
- Major earthquakes (M 7 to 8)
- Great earthquakes (M > 8)

1.13.3 Based on origin

- Tectonic earthquakes
- Plutonic earthquakes
- Explosions
- Collapse earthquakes
- Volcanic earthquakes
- Reservoir induced earthquakes

SEISMIC ZONING MAP OF INDIA – BACKGROUND, BASIS, ZONE FACTOR

India is seismically active and has experienced many earthquakes in the past. Fig. 2 and Table 3 present some of the past earthquakes and their effects on Indian soil. More than 60 % of the country is considered to be in seismically active regions. Based on the past experience, geologic activities, presence of active faults and closeness to plate boundary, the country is divided in to 4 zones - Zone 2 to Zone 5. Zone 2 is seismically least active and zone 5 is seismically most active. In seismically very active zone, the frequency of big earthquake and possibility of strong shaking are more. Over years, Indian codal provisions are evolved and the following are the important modifications in the recent version of IS 1893 – Part 1 – 2002.

Major modifications in the recent I S code

1. Zone I is merged with Zone II.
2. Values of seismic zone factors are changed considering MCE & service life of structure.
3. Response spectra are specified for THREE types of soils - Rock & Hard Soil, Medium Soil and Soft Soil.
4. Empirical equations for time period of multi storey buildings are revised.

Fig. 14 presents the map of India with different seismic zones. Karnataka is seismically quite stable and most part of it is in Zone 2. Only coastal Karnataka and some parts in north are in Zone 3. Table 5 provides the details of zone factor in different zones. It can be seen that the zone factor is 3.6 times bigger in Zone 5 than in Zone 2. Hence, the horizontal force is 3.6 times bigger in Zone 5 than in seismically least active places.

References

• *Chen W H and Scawthorn C (2003) : Earthquake Engg Handbook, C R C Press*

• *Das B. M. (1993): Principles of Soil Dynamics, Elsevier*

• *Day R. W. (2003) : Geotechnical Earthquake Engineering Handbook, Mc Graw Hill*

• *Ishihara. K. (1996) : Soil Behavior in Earthquake Geotechnics, Clarendon Press, Oxford*

• *Krammer S. L. (1996): Geotechnical Earthquake Engineering, Prentice Hall*

• *Okamoto S. (1984) : Introduction to Earthquake Engg, University of Tokyo Press*

• *Pankaj Agarwal and Manish Shirkande (2006): Earthquake Resistant Design of Structures, Prentice Hall of India*

• *Earthquake Tips, NICEE, IIT, Kanpur,* www.**nicee**.org/EQ**Tips**.php

STRUCTURAL DYNAMICS

BASICS:

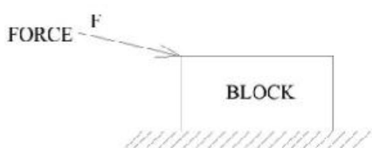
- Real-life structures are subjected to loads which vary with time.
- Except self weight of the structure, all other loads vary with time. In many cases, this variation of the load is small, hence static analysis is sufficient. However, in case of off-shore structures (oil rigs), high rise buildings subjected to lateral loads (wind, earth quake) dynamic effects of the load must be explored for knowing the exact safety and reliability of the structure.

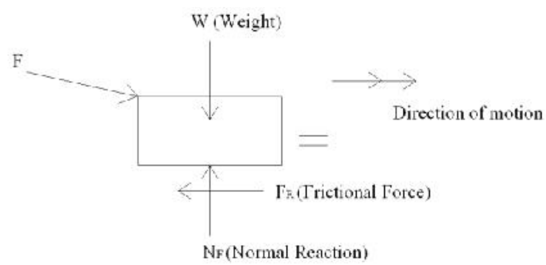
Comparison between static and dynamic analysis:

Static analysis	Dynamic analysis
Loads are constant (magnitude, direction and point of application), hence time invariant.	Loads are varying with time, hence analysis depends on time also.
Static equilibrium is applicable.	Dynamic equilibrium is applicable.
Motion does not occur.	The characteristic of motion in the form of displacement, velocity and acceleration become important parameters.

D'ALEMBERT'S PRINCIPLE:

Consider a block resting as a horizontal surface. Let it be subjected to a force as shown in figure and set to motion. The FBD of the block is as shown.





For the system of forces acting on FBD, we can find a single force called Resultant Force. By Newton's Second Law of Motion, this resultant force must be equal to $R_F = ma$, where m is the mass of the block = $^W/_g$ and a is acceleration of the block.

To the FBD, if we now add a force, which is equal to R_F in magnitude and opposite in sense, as shown above then this diagram will be in dynamic equilibrium. This force F_i is an imaginary force called as inertial force or reverse effective force.

The principle of adding F_i to FBD is called as D'Alembert's principle.

Some Definitions:

Vibration and oscillation: If motion of the structure is oscillating (pendulum) or reciprocatory along with deformation of the structure, it is termed as VIBRATION. In case there is no deformation which implies only rigid body motion, it is termed as OSCILLATION.

Free vibration: Vibration of a system which is initiated by a force which is subsequently withdrawn. Hence this vibration occurs without the external force.

Forced Vibration: If the external force is also involved during vibration, then it is forced vibration.

Damping: All real life structures, when subjected to vibration resist it. Due to this the amplitude of the vibration gradually, reduces with respect to time. In case of free vibration, the motion is damped out eventually. Damping forces depend on a number of factors and it is very difficult to quantify them. The commonly used representation is viscous damping wherein damping force is expressed as $F_0=C\dot{x}$ where $\dot{x}$ = velocity and C=damping constant.

Degree of Freedom: It is very well known that any mass can have six displacement Components (3 translations and 3 rotations). In most systems, some of these displacements are restrained. The number of possible displacement components is called as Degree of Freedom (DoF). Hence DOF also represents minimum number of coordinate systems required to denote the position of the mass at any instant of time.
An overhead tank is considered as an example. This can be modeled as a cantilever column with concentrated mass at top. If we want axial vibration, then only one coordinate (y) is sufficient. If only the flexural deformation is required then also only one co-ordinate (x) is required. If both are required, then two coordinates are required.
Depending upon the co-ordinates to describe the motion, we have

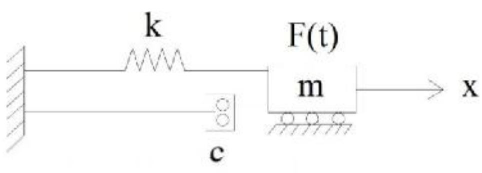
1. Single degree of freedom system (SDoF).
2. Multiple degree of freedom (MDoF).
3. Continuous system.

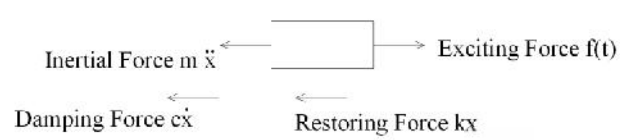
Free Vibration of SDoF: An SDoF is one which needs only one co-ordinate to describe the motion. The single bay single storey rigid frame is taken as SDoF based on assumptions.

i) Mass of columns are small compared to the mass of the beam. Hence neglected.
ii) Girder is infinitely rigid structure, hence it does not deform and hence the stiffness is provided only by column.
When this frame vibrates due to lateral load in horizontal direction, the force acting are inertial force, (2) damping force, (3) restoring force, and (4) External force.

If the external force is removed after initial disturbance, the free vibration occurs. Further it will be treated as free damped vibration, if damping is present and if damping is not present it is called free undamped vibration.

An SDoF is represented as shown in Figure.





For this system, only one coordinate x is required (translation). Displacement is x, velocity is $\dot{x}\left(\frac{dx}{dt}\right)$ and $\ddot{x}\left(\frac{d^2x}{dt^2}\right)$ is acceleration.

The inertial force hence is $m\ddot{x}$. The damping force is $c\dot{x}$ and spring force is kx.

Using D'Alembert's principle, by dynamic equilibrium.

$$m\ddot{x} + c\dot{x} + kx - f(t) = 0;$$

$$m\ddot{x} + c\dot{x} + kx = f(t); \rightarrow (1)$$

This is 2nd order differential equation. The solution of this equation gives response of an SDoF system.

Free Undamped Vibration: In this case, $f(t)$ is zero and C is zero (because no damping).

Hence (1) is $m\ddot{x} + kx = 0;\ \ddot{x} + \frac{k}{m}x = 0.$

Let $\sqrt{k/m} = p;\qquad \ddot{x} + p^2 x = 0 \rightarrow (2)$

The solution in the above equation is of the form $x = Ae^{\lambda t} \rightarrow (3)$ Then $\dot{x} = Ae^{\lambda t}(\lambda)$

$\ddot{x} = Ae^{\lambda t}(\lambda^2)$

Therefore, equation (2) is

$\lambda^2 Ae^{\lambda t} + p^2 Ae^{\lambda t} = 0.$

$\lambda^2 + p^2 = 0.$

$\lambda^2 = -p^2 \Rightarrow \lambda = \pm ip$.

Where $i = \sqrt{-1}$; Hence (3) is $x = A_1 e^{ipt} + A_2 e^{-ipt}$.

$x = A_1\{\cos pt + i\sin pt\} + A_2\{\cos pt - i\sin pt\} \rightarrow (4)$

On rearranging,

$x = c_1 \cos pt + c_2 \sin pt \rightarrow (5).$

c_1 and c_2 are constants. Since cosine and sine functions are periodic functions, motion defined by x will also be periodic (motion repeats itself after certain interval of time).

T $\rightarrow$ time period when motion completes one complete rotation.

Since $p = 2\pi \Rightarrow T = \frac{2\pi}{p}, T = 2\pi\sqrt{\frac{m}{k}} \rightarrow (6)$

T is called time period of an undamped free vibration system. Reciprocal of T is the frequency which is nothing but number of times motion repeats itself in one second.

This reciprocal is represented as f and it is natural frequency of the system.

$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow (7)$ Hz (cycle/s)

Since $p = \frac{2\pi}{T} = 2\pi f$.

p is called circular frequency or angular frequency of vibration (Rad/s)

Equation (5) is a harmonic motion c_1 and c_2 can be determined from certain initial conditions.

For example: if at $t = 0, x = x_0$ and $\dot{x} = \dot{x}_0$,

From (5) $\dot{x} = c_1 p(-\sin pt) + c_2 p(\cos pt)$.

At $t = 0$, $\dot{x} = \dot{x}_0 = c_1 p(-\sin 0) + c_2 p(\cos 0)$.

$\dot{x}_0 = c_2 p \Rightarrow \frac{\dot{x}_0}{p} = c_2$

From (5) At t=0; $x = x_0 = c_1 \cos 0 + c_2 \sin 0$.

$x_0 = c_1 \Longrightarrow c_1 = x_0$.

Therefore (5) is $x = x_0 \cos pt + \frac{\dot{x}_0}{p} \sin pt$ → (9)

Further manipulation is done by multiplying and dividing the terms on RHS of (9) by a factor A. then

$$x = A\left\{\frac{x_0}{A} \cos pt + \frac{\dot{x}_0/p}{A} \sin pt\right\}$$

A is defined by geometry as $A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{p}\right)^2}$.

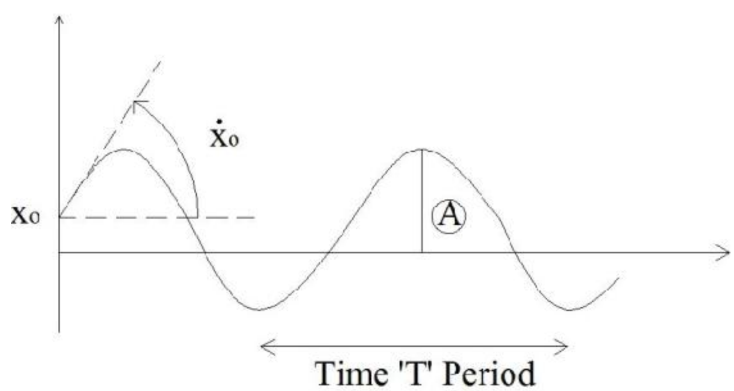
$\sin \beta = \frac{\dot{x}_0/p}{A}$; $\cos \beta = \frac{x_0}{A}$;

Then $x = A\{\cos \beta \cos pt + \sin \beta \sin pt\}$

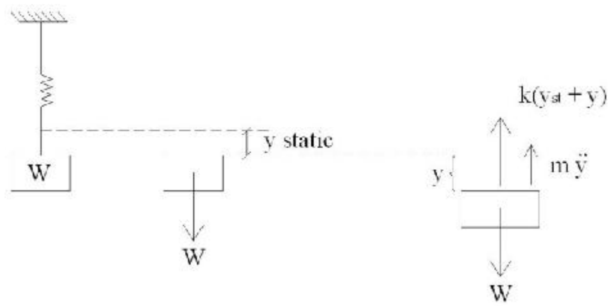
$x = A\{\cos(pt - \beta)\}$→(10).

A in the above equation is the amplitude of motion. Angle β is called phase angle. Equation (10) is harmonic in nature. β which is the phase angle is computed as

$\frac{\sin \beta}{\cos \beta} = \tan \beta = \frac{\dot{x}_0}{p x_0}$.



Problem 1: Weight of 15 N is vertically suspended by a spring of stiffness k=2 N/mm. Determine natural frequency of free vibration of weight.



$$y_{static} = {}^{w}/_{k}.$$

$$m\ddot{y} + k(y_{st} + y) - w = 0.$$

$$^{w}/_{g}\, \ddot{y} + k(y_{st} + y) - w = 0.$$

$$^{w}/_{g}\, \ddot{y} + k\left({}^{w}/_{k} + y\right) - w = 0.$$

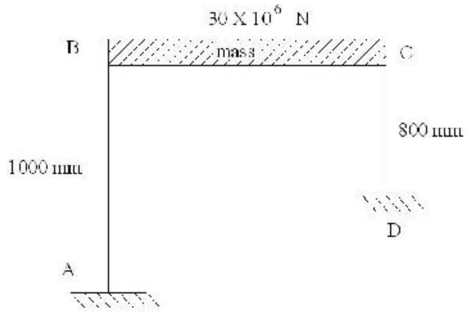
$$^{w}/_{g}\, \ddot{y} + ky = 0. \qquad \sqrt{\frac{k}{m}} = p$$

$$\ddot{y} + py = 0. \qquad p = \sqrt{\frac{k}{\left(\frac{w}{g}\right)}} = \sqrt{\frac{kg}{w}}.$$

$$p = \sqrt{\frac{kg}{w}} = \sqrt{\frac{2(9810)}{15}} = 36.166\ Rad/sec.$$

$$f = \frac{1}{2\pi} p = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 5.76\ Hz\ (cps).$$

Problem 2: Calculate the natural angular frequency of the frame shown in figure. Compute also natural period of vibration. If the initial displacement is 25 mm and initial velocity is 25 mm/s what is the amplitude and displacement @t =1s.



In this case, the restoring force in the form of spring force is provided by AB and CD which are columns. The equivalent stiffness is computed on the basis that the spring actions of the two columns are in parallel.

 $k_{eq} = k_{AB} + k_{CD}$.

$= \frac{12\,EI}{1000^3} + \frac{12\,EI}{800^3} =$ 1063125 N/mm.

 $Natural\ circular\ frequency = \sqrt{\frac{k_{eq}}{m}}.\ p = \sqrt{\frac{k_{eq}g}{w}}$.

 $p = \sqrt{\frac{1063125\times9810}{30\times10^6}} = 18.645 Rad/sec.$

Natural frequency $f = \frac{1}{2\pi}p = 2.967$ cycles/s.

Natural period = $T = 0.337$ secs.

$x = A\cos(pt - \beta).$

Given At $t = 0; x_0 = 25\,mm; \dot{x}_0 = 25\,mm/s$

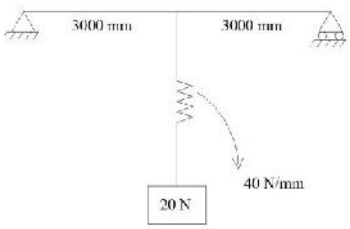
$A = \sqrt{25^2 + \left(\frac{25}{18.645}\right)^2} = 25.03\,mm.$

$\beta = \tan^{-1}\left(\frac{\dot{x}_0/p}{x_0}\right) = \tan^{-1}\left(\frac{25/18.645}{25}\right) =$0.0535 Radians.

At t=1s,

$x = 25.03\cos(18.645t - 0.0535) = 24.2\,mm.$

Problem 3: Find the natural frequency of the system shown. The mass of the beam is negligible in comparison to the suspended mass. E=2.1 x 10^5 N/mm^2.



The beam has elastic behavior and hence has spring action. Further spring attached to the mass also has a restoring force. The deformation of the mass at the centre is equal to deformation of beam at midspan and that of attached spring.

Central deflection in a beam carrying a single concentrated load is $\delta = \frac{pl^3}{48\,EI}$

 For $\delta = 1$; p is the stiffness which is $k = \frac{48EI}{l^3}$;

Hence the system is considered to contain two springs k_1(beam), k_2(direct spring) which are connected in series.

In a series connection, the equivalent stiffness k_{eq} is

$$1/k_{eq} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$$

$$I = \frac{1}{12}(100)(150)^3 = 28125000\ mm^4.$$

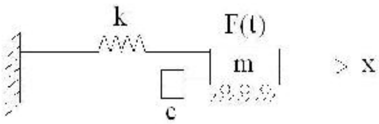
$$EI = 5.90625 \times 10^{12} N - mm^2$$

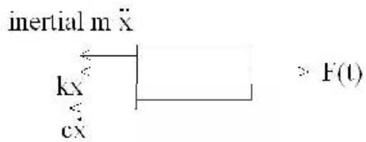
$$k_{eq} = \frac{1312.5(40)}{1312.5+40} = 38.82 \text{ N/mm.}$$

$$p = \sqrt{\frac{k}{m}} = 137.99 \text{ Rad/sec}$$

$$f = \frac{1}{2\pi} p = 21.962 \text{ Hz.}$$

Free damped vibration of SDF system:





In case of free vibration, F(t)=0; $m\ddot{x} + c\dot{x} + kx = 0 \longrightarrow (1).$

This is a d.e of 2nd order the general solution of which is $x = Ae^{\lambda t} \longrightarrow (2).$

$\therefore\ \dot{x} = \frac{dx}{dt} = A\lambda e^{\lambda t}; \ddot{x} = \frac{d^2x}{dt^2} = A\lambda^2 e^{\lambda t};$

Putting, these values in (1) and dividing by m throughout,

$A\lambda^2 e^{\lambda t} + \frac{c}{m}\lambda e^{\lambda t} + \frac{k}{m}Ae^{\lambda t} = 0.$

$Ae^{\lambda t}\left\{\lambda^2 + \lambda c/m + k/m\right\} = 0$

$\lambda^2 + \lambda\frac{c}{m} + \frac{k}{m} = 0$

Let $\sqrt{\frac{k}{m}} = p$; and $\frac{c}{2m} = n$;

$\lambda^2 + 2n\lambda + p^2 = 0 \longrightarrow (3);$

p → constant for a given system; n → $\frac{c}{2m}$ → c damping constant, m is also a constant.

Hence (3) is quadratic equation in λ and its solution is

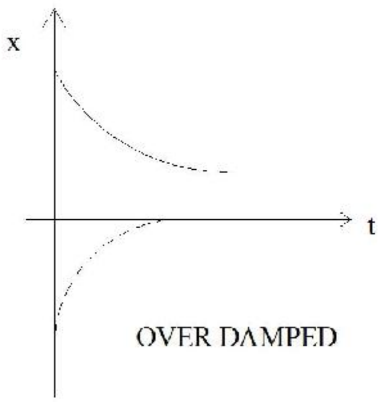
$\lambda_{1,2} = \frac{-2n\pm\sqrt{(2n)^2-4p^2}}{2} = -n \pm \frac{\sqrt{4n^2-4p^2}}{2}$

$\lambda_{1,2} = -n \pm \sqrt{n^2 - p^2} \longrightarrow (4)$

The net value of $\lambda_{1,2}$ depends on values of n and p. Hence following cases are possible.

<u>Case 1:</u> n>p; in this case, $\lambda_{1,2}$ both are real but negative. The system is said to be over damped. The motion equation is $x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \longrightarrow (5)$

Both λ_1 and λ_2, are negative which implies that motion decays exponentially with time. Hence in this case, there is no periodicity (or harmonic motion) but the damping is so huge, that the system just comes back slowly equilibrium from displaced position.



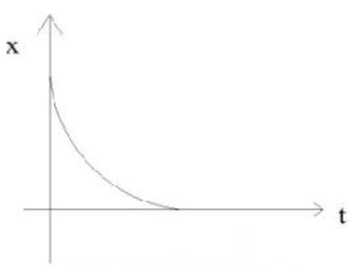
OVER DAMPED

<u>Case 2:</u> n=p;

In this case, in equation (4) the term under the square root vanishes. Hence $\lambda_{1,2}$ are real, negative and equal.

∴ Solution is $x = A_1 e^{\lambda t} + A_2 e^{\lambda t} \longrightarrow (6)$.

Even in this case, there is no oscillatory motion. The damping in the system is enough to bring it back to equilibrium. System is said to be **Critically Damped**.



CRITICAL DAMPED

Since, n = p, $\frac{c}{2m} = p$, c = 2mp.

Since this represents damping constant corresponding to critical system it is expressed as C_c=2mp.

Case 3: n<p;

Since n is less than p, the term under the square root becomes negative. Hence the roots are complex conjugate roots.

 $\lambda_{1,2} = -n \pm i\sqrt{p^2 - n^2}$.

 motion equation is $x = A_1 e^{\left\{-n+i\sqrt{p^2-n^2}\right\}t} + A_2 e^{\left\{-n-i\sqrt{p^2-n^2}\right\}t}$

OR $x = A_1 e^{-nt} e^{it\sqrt{p^2-n^2}} + A_2 e^{-nt} e^{-it\sqrt{p^2-n^2}}$

$$x = e^{-nt}\left[A_1 e^{it\sqrt{p^2-n^2}} + A_2 e^{-it\sqrt{p^2-n^2}}\right]$$

Since, $e^{\pm it\sqrt{p^2-n^2}} = \cos\sqrt{p^2-n^2}\,t \pm i\,\sin\sqrt{p^2-n^2}\,t$

$$x = e^{-nt}\left[(A_1 + A_2)\cos\sqrt{p^2-n^2}\,t + i(A_1 - A_2)\sin\sqrt{p^2-n^2}\,t\right]$$

Let $(A_1 + A_2) = c_1$; and $i(A_1 - A_2) = c_2$

$$x = e^{-nt}\left[c_1 \cos\sqrt{p^2-n^2}\,t + c_2 \sin\sqrt{p^2-n^2}\,t\right] \longrightarrow (7)$$

c_1 and c_2 are constants to be calculated from known initial conditions.

At t=0, $x=x_0$ and velocity = $\dot{x}_0$. Using these conditions, c_1 and c_2 can be computed.

At t=0;

$x=x_0$

$$x = x_0 = 1[c_1 \cos 0 + c_2 \sin 0] = 0 \Rightarrow c_1 = 0$$

$$\dot{x} = e^{-nt}\left[-c_1\sqrt{p^2-n^2}\sin\sqrt{p^2-n^2}\,t + c_2\sqrt{p^2-n^2}\cos\sqrt{p^2-n^2}\,t\right] -$$

$$ne^{-nt}\left\{c_1 \cos\sqrt{p^2-n^2}\,t + c_2 \sin\sqrt{p^2-n^2}\,t\right\}$$

At t=0; $\dot{x} = \dot{x}_0 = 1\left[x_0(0) + c_2\sqrt{p^2-n^2}\right] - n[x_0(1) + c_2(0)]$

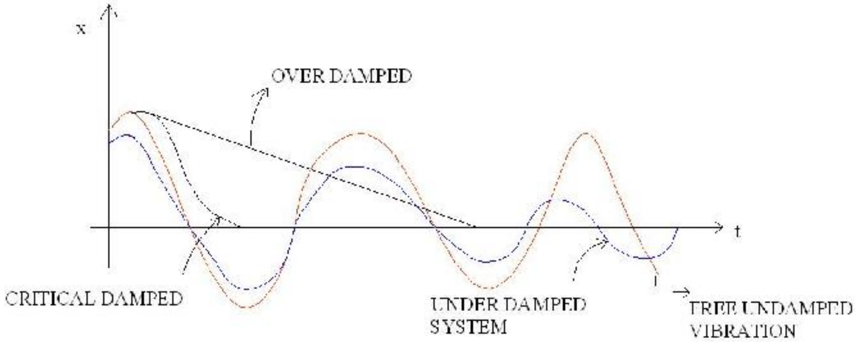
$$\dot{x}_0 = c_2\sqrt{p^2-n^2} - nx_0$$

$$\frac{\dot{x}_0 + nx_0}{\sqrt{p^2-n^2}} = c_2\ ;$$

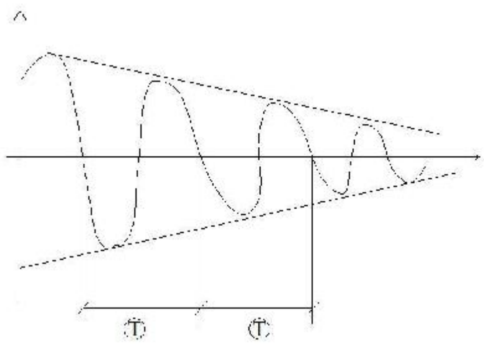
 $x = e^{-nt}\left[x_0 cos\sqrt{p^2-n^2}\,t + \left(\frac{nx_0+\dot{x}_0}{\sqrt{p^2-n^2}}\right)sin\sqrt{p^2-n^2}\,t\right] \longrightarrow (8).$

Equation (8) describes motion in undamped case.

The amplitude of vibration goes down gradually in an exponential manner.



The variation of the displacement under damped free vibration system is as shown. The amplitude decreases in an exponential manner.



The damped angular frequency P_d is written as $p_d = \sqrt{p^2 - n^2}$.

 Time period = $\frac{2\pi}{p_d} = \frac{2\pi}{\sqrt{p^2-n^2}}$

$$T_d = \frac{2\pi}{p\sqrt{1-(n/p)^2}} = \frac{2\pi}{p}\left(\frac{1}{\sqrt{1-(n/p)^2}}\right) \longrightarrow (9)$$

$\frac{1}{\left(\sqrt{1-(n/p)^2}\right)}$ will be greater than one. This implies $T_d > \frac{2\pi}{p}$ (which is natural undamped time period). This means that damped natural time periods is more that of undamped period. However, this increase in time period is very small and hence for all practical purposes, it is assumed that a small viscous damping will not affect time period of vibration.

Problem 1: A platform of weight 18 KN is being supported by four equal columns which are clamped to the foundation. Experimentally, it has been computed that a static force 5KN applied horizontally, to the platform produces a displacement of 2.5 mm. It is estimated that the damping in the structure is of the order of 5% of critical damping. Compute the following:

a) Undamped natural frequency.

b) Damping coefficient.

c) Logarithmic decrement.

d) No of cycles and time required for amplitude of motion to be reduced from an initial value of 2.5 mm to 0.25 mm.

Stiffness is force/unit disp = $\frac{5x10^3}{2.5}$

= 2000 n/mm.

Undamped natural frequency = $\sqrt{\frac{k}{m}}$

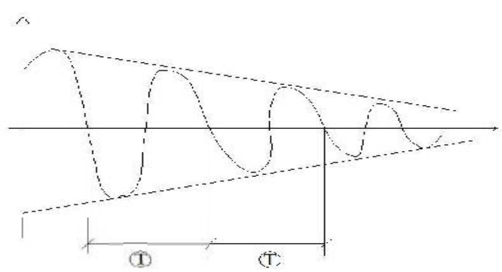
$\sqrt{\frac{2000 \times 9810}{18 \times 10^3}}$ =33.01 Rad/sec.

Critical damping coefficient $c_c = 2\sqrt{km} = 2\sqrt{2000 \times \frac{18\times10^3}{9.81\times10^3}} = 121.16\ NS/mm.$

 actual damping = 5% of C_c = $\frac{5}{100} \times 121.16 = 6.06\ NS/mm.$

Logarithmic decrement:

For a free vibration under damped SDoF system, the osciallatory motion is as shown in the figure. With the increase in time, there is a gradual decrease in amplitude. This delay of amplitude is expressed by using logarithmic decrement 'δ'. δ is the natural logarithm of the ration of any two successive peak amplitude say x_1 and x_2.



 $\delta = ln\left(\frac{x_1}{x_2}\right)$.

At any instant say t_1, $x_1 = e^{-nt_1}\left\{c_1 \cos\sqrt{p^2-n^2}\, t_1 + c_2 \sin\sqrt{p^2-n^2}\, t_1\right\}$.

Next positive amplitude x_2 will occur at time $t = t_2 + T = t_1 + \frac{2\pi}{\sqrt{p^2-n^2}}$

$$□\ x_2 = e^{-n\left[t_1 + \frac{2\pi}{\sqrt{p^2-n^2}}\right]}\left\{c_1 \cos\sqrt{p^2-n^2}\left[t_1 + \frac{2\pi}{\sqrt{p^2-n^2}}\right] + c_2 \sin\sqrt{p^2-n^2}\left[t_1 + \frac{2\pi}{\sqrt{p^2-n^2}}\right]\right\}.$$

Upon simplification $\frac{x_1}{x_2} = \frac{1}{e^{\frac{-2\pi n}{\sqrt{p^2-n^2}}}}$, taking logarithm on either side $ln\left(\frac{x_1}{x_2}\right) = \delta = \frac{2\pi n}{\sqrt{p^2-n^2}}$

i.e., $\delta = \frac{2\pi^{n}/_{p}}{\sqrt{^{p^2}/_{p^2} - {^{n^2}/_{p^2}}}}$

Let $^{n}/_{p} = \xi$, then $\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$.

Since, ξ is very small value we write $\delta = 2\pi\xi$.

Therefore $\xi = \frac{1}{2\pi}\, ln\left(\frac{x_1}{x_2}\right)$

By using this ratio, critical damping is computed.

The ratio between the first amplitude and k^{th} amplitude is $\frac{x_0}{x_k} = \frac{x_0}{x_1}\frac{x_1}{x_2}\dots\dots\frac{x_{k-1}}{x_k}$

Taking logarithm on either side we write $ln\left(\frac{x_0}{x_k}\right) = k\delta$.

In this case, $\delta = 2\pi\xi = 0.314$.

Therefore, $ln\left(\frac{2.5}{0.25}\right) = k(0.314)$

$k = 7.33$ □ $8\ Cycles$

$p_d = 32.96\ Rad/s$

$T_d = 0.19\ Sec$

Time for 8 Cycles = 1.52 Sec.

Critical damping is defined as the least value of damping for which the system does not osciallate and which the system does not oscillate. When disturbed initially, it simply will return to the equilibrium position after an elapse of time.

Problem 2: A vibrating system consisting of a weight of w=50 N and a spring with stiffness of 4N/mm is viscously damped. The ration of two successive amplitudes is 1:0.85 compute

a) natural frequency (undamped) of the system.

b) logarithmetic decrement

c) damping ratio

d) the damping co.eff and

e) damped natural frequency

undamped natural frequency $p = \sqrt{\frac{k}{m}}$; $m = \frac{50}{9810}$; $K = 4$ *N/mm*

$p = \sqrt{\frac{4}{50/9810}} = 28.01$ *Rad/sec*.

$f = \frac{p}{2\pi} = 4.46$ cycles/sec.

Logarithmic decrement $\delta = ln\left(\frac{x_1}{x_2}\right) = ln\left(\frac{1}{0.85}\right)$; $\delta = 0.162$.

Damping ratio: ξ

$\delta = 2\pi\xi$; $\xi = \frac{\delta}{2\pi} = \frac{0.162}{2(\pi)} = 0.026$.

Damping co-efficient $= \xi = \frac{c}{2\sqrt{km}}$;

$c = 2\xi\sqrt{km} = 2(0.026)\sqrt{4\left(\frac{50}{9810}\right)} = 0.0074\times10^{-3}$ *NS/mm*

Natural damped frequency p_d.

$p_d = p\sqrt{1-\xi^2} = 28.01\sqrt{1-0.026^2}$; $p_d = 28$ *Rad/Sec*.

Forced vibration of SD_oF system:

Consider a SD_oF system under forced vibration as illustrated.

$m\ddot{x} + c\dot{x} + kx = F(t) \rightarrow (1)$

This is a 2nd order ODE for which solution is of the form $x = CF + PI \rightarrow (2)$.

CF-> Complimentary function which is the solution obtained by taking RHS in equation 1 as zero. Then, the equation obtained is nothing but free vibration case.

PI-> This is the solution obtained which has direct linkage to the forcing function F(t).

F(t) which is the dynamic load varies with time. If F(t) is a harmonic function w.r.t time say $F(t)=F_0 \sin wt$, where F_0->Amplitude, W->Angular frequency, then equation (1) is $m\ddot{x} + c\dot{x} + kx = F_0 \sin wt \rightarrow (3)$.

Using the terms $n = \frac{c}{2m}$ and $p^2 = {}^{k}/_{m}$.

$\ddot{x} + 2n\dot{x} + p^2 x = \frac{F_0}{m} \sin wt \rightarrow (4)$ the solution then is $x = (x)_{CF} + (x)_{PI} \rightarrow (5)$, where

$x_{(cF)=e^{-nt}\{c_1 \cos\sqrt{p^2-n^2}t + c_2 \sin\sqrt{p^2-n^2}t\}} \rightarrow (6)$.

The solution as PI is of the form $(x)_{PI} = A\cos wt + B\sin wt \rightarrow (7)$.

Therefore $(\dot{x})_{PI} = -A\sin wt(w) + B\cos wt(w)$

$(\ddot{x})_{PI} = -A\,w^2 \cos wt - Bw^2 \sin wt$. Substitute these values in equation (4)

$$-Aw^2 \cos wt - Bw^2 \sin wt + 2n\{-Aw\sin wt + Bw\cos wt\} + p^2\{A\cos wt + B\sin wt\} = \frac{F_0}{m}\sin wt.$$

$$\cos wt\{-AW^2 + 2nBw + Ap^2\} + \sin wt\{-Bw^2 - 2nAw + Bp^2\} = \frac{F_0}{m}\sin wt$$

Comparing coefficients, $-Aw^2 + 2nBw + Ap^2 = 0$

$$-Bw^2 - 2nAw + Bp^2 = \frac{F_0}{m}$$

$$A(p^2 - w^2) + 2nBw = 0 \quad A = \frac{-2nBw}{(p^2-w^2)}$$

$$B(p^2 - w^2) - 2nAw = \frac{F_0}{m}$$

$$B(p^2 - w^2) + 2nw\left\{\frac{2nBw}{p^2-w^2}\right\} = \frac{F_0}{m} .$$

$$B\left[(p^2 - w^2) + \frac{(2nw)^2}{(p^2-w^2)}\right] = \frac{F_0}{m}.$$

$$B = \frac{{}^{F_0}/_{m}(p^2-w^2)}{(p^2-w^2)+(2nw)^2}$$

$$A = \frac{-2nw}{(p^2-w^2)}\left\{\frac{+\frac{F_0}{m}(p^2-w^2)}{(p^2-w^2)^2+(2nw)^2}\right\}.$$

By using trignometrical equations of $\cos\phi = \frac{(p^2-w^2)}{\sqrt{(p^2-w^2)^2+(2nw)^2}}$ and $\sin\phi = \frac{2nw}{\sqrt{(p^2-w^2)^2+(2nw)^2}}$

$$x_{PI} = \left\{\frac{-\frac{F_0}{m}(2nw)}{(p^2-w^2)^2+(2nw)^2}\right\}\cos wt + \left\{\frac{-\frac{F_0}{m}(p^2-w^2)}{(p^2-w^2)^2+(2nw)^2}\right\}\sin wt$$

$$x_{PI} = \frac{\frac{F_0}{m}}{\sqrt{(p^2-w^2)^2+(2nw)^2}}\{\sin wt \ \cos\phi - \cos wt \ \sin\phi\}$$

$$x_{PI} = \frac{\frac{F_0}{m}}{\sqrt{(p^2-w^2)^2+(2nw)^2}}\sin(wt-\phi).$$

$x_{(cF)}$ corresponds to free damped vibration which decays with elapse of time.

$x_{(PI)}$ corresponds to motion relating to the forcing function. Hence it will have the same frequency as the force. The actual motion is superposition of $x_{(cf)}$ and $x_{(PI)}$. Since $x_{(cf)}$ is a decaying component, it is termed as transient vibration, $x_{(PI)}$ is termed as steady state vibration.

Eqn (9) is rewritten as $x = \frac{\frac{F_0}{mp^2}}{\sqrt{\left(1-{}^{w}/_{p}\right)^2+\left(\frac{2n}{p}\cdot\frac{w}{p}\right)^2}}\sin(wt - \phi)$.

${}^{w}/_{p} = \eta$ = tuning factor = resonant frequency ratio.

$\frac{\eta}{p} = \xi$ =damping ratio.

$mp^2 = k$;

$\therefore x = \frac{\left({}^{F_0}/_{k}\right)\sin(wt-\phi)}{\sqrt{(1-\eta^2)^3+(2\eta\xi)^2}}$ → (10).

${}^{F_0}/_{k}$ =static displacement $= \delta_{st}$.

$$x = \frac{\delta_{st}}{\sqrt{(1-\eta^2)^3+(2\eta\xi)^2}}\sin(wt - \theta).$$

Max. amplitude of motion hence is $x_{max} = \frac{\delta_{st}}{\sqrt{(1-\eta^2)^2+(2\eta\xi)^2}}$.

The ratio of dynamic displacement (motion) with that of displacement of statics is known as Dynamic Load Factor (DLF).

$DLF = \frac{x}{\delta_{st}} = \frac{\sin(wt-\phi)}{\sqrt{(1-\eta^2)^2+(2n\xi)^2}}$.

Maximum value of $DLF = \frac{1}{\sqrt{(1-\eta^2)^2+(2n\xi)^2}}$.

This maximum value of DLF known as magnification factor μ is $\frac{1}{\sqrt{(1-\eta^2)^2+(2n\xi)^2}}$.

In order to get the maximum value of μ, we compute $\frac{d\mu}{d\eta} = \frac{d}{d\eta}\left\{\frac{1}{\sqrt{(1-\eta^2)^2+(2n\xi)^2}}\right\}$

$$\frac{-2\eta\ 2\,(1-\eta^2)+4\xi^2(2\eta)}{-2[(1-\eta^2)^2+(2\eta\xi)^2]^{3/2}} = 0$$

$$-4n(1-\eta^2) + 8n\xi^2 = 0$$

Or $\eta = \sqrt{1-2\xi^2}$ → (11).

ξ being a small quantity, maximum value of μ obtained when frequency of external force is nearly equal to the frequency of the system in free vibration, μ increases rapidly. This condition is called Resonance.

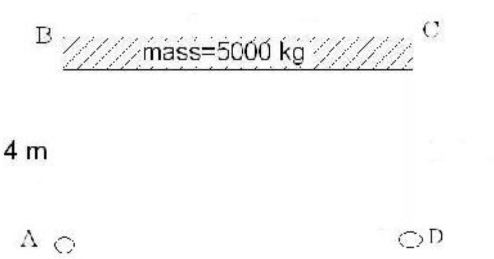
$$\phi = \tan^{-1}\left(\frac{2\eta w}{p^2-w^2}\right) = \tan^{-1}\left(\frac{2^{n}/_{p}{}^{w}/_{p}}{(1-(^{w}/_{p})^2)}\right)$$

$$\phi = \tan^{-1}\left(\frac{2\xi\eta}{(1-\eta^2)}\right) \rightarrow (12)$$

Conclusions:

1. The free vibration part is transient and vanishes, forced part persists.
2. With increase of ξ, magnification factor reduces.
3. The magnitude of the maximum value of magnification factor is very sensitive to the value of ξ.
4. Steady state vibration is independent of initial conditions of the system.

Problem 1: A steel rigid frame (one bay one storey) having hinged supports, carries a rotating machine. This escorts a horizontal force at girder level in the form of "50000 sin 11 t" N assuming 4% critical damping, what is steady state amplitude of vibration? I for columns $=1500\times10^{-7}$m^4, E$=21\times10^{10}$N/m^2.



Stiffness for each column = $3EI/l^3$.

Stiffness = $2(3EI/l^3)$

$\therefore$K= 2953125 N/m.

Natural frequency p = $\sqrt{\frac{k}{m}} = 24.3\,Rad/s.$

ξ=0.04

$\eta = 0.453$

Therefore $x_{max} = \frac{F_0/k}{\sqrt{(1-\eta^2)^2+(2\eta\xi)^2}} = 0.0213m = 21.33\,mm.$

EARTHQUAKE RESISTANT DESIGN OF MASONRY BUILDINGS

Introduction

Masonry construction is the oldest and most common building technique, together with timber construction. The word “masonry” actually encompasses techniques which may differ substantially depending on type and shape of materials and construction methods. In general, masonry may be defined as a structural assemblage of masonry units (such as stones, bricks and blocks) with a binding material known as mortar. A vertical two-dimensional structure of such an assemblage is known as masonry wall. The walls of a masonry building and the building itself are designed to be stable, strong and durable to withstand a combination of design loads.

The basic advantage of masonry construction is that it is possible to use the same element to perform a variety of functions, which in a framed building, for example, have to be provided for separately, with consequent complication in detailed construction. Thus masonry may, simultaneously, provide structure, subdivision of space, thermal and acoustic insulation as well as fire and weather protection. As a material, it is relatively economical, durable and produces external wall finishes of acceptable appearance. Masonry construction is flexible in terms of building layout and can be constructed without very large capital expenditure on the part of the builder.

In India, at present, IS-1905 (1987, reaffirmed 1998) is the code of practice for “Structural Use of Un-reinforced Masonry”. A detailed hand book on Masonry Design and Construction is published by Bureau of Indian Standards in the form of SP-20 (S&T, 1991). An IS code for Structural Use of Reinforced Masonry is under preparation.

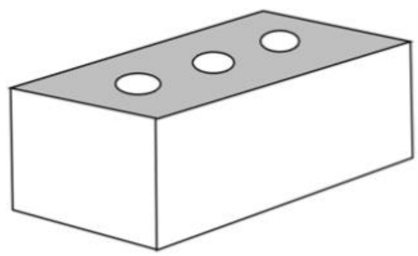
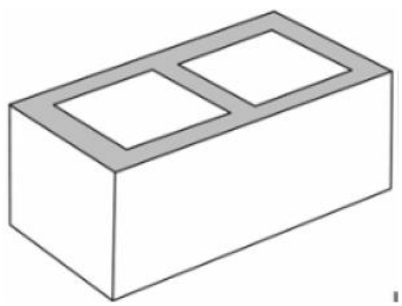
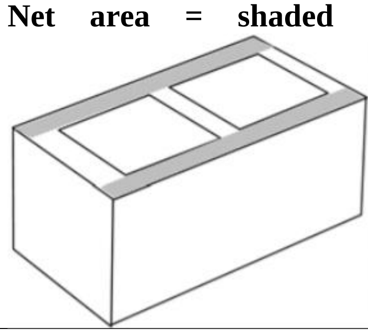
There are some guidelines for construction of reinforced masonry in IS-4326 (1993, reaffirmed 1998), mainly for earthquake resistant design and construction of masonry buildings. Guidelines for improving earthquake resistance of low-strength masonry buildings are covered separately in IS-13828 (1993, reaffirmed 1998).

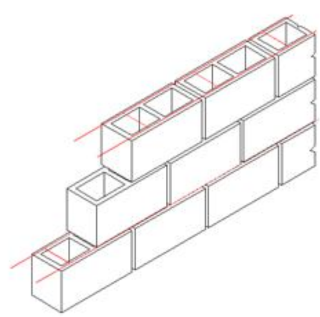
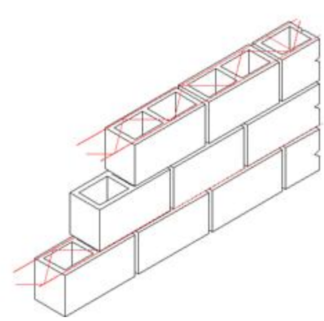
This chapter contains the following;

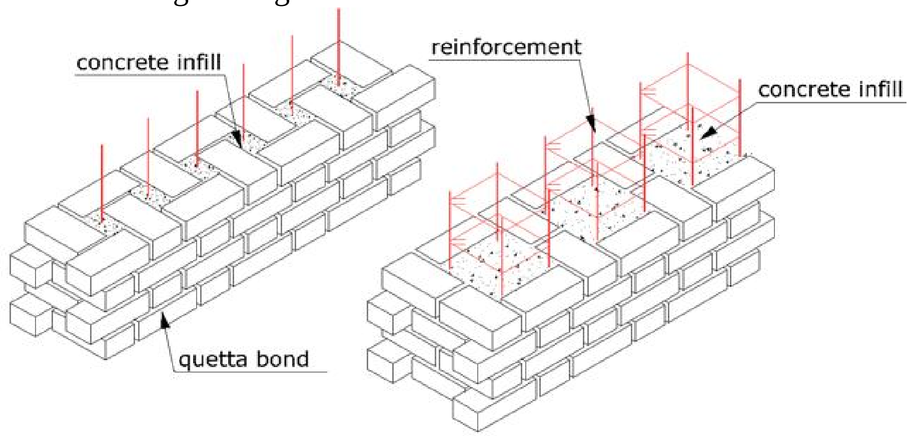
1. Terminologies in structural masonry
2. Basics of design of load bearing masonry
3. Concepts for reinforced masonry and earthquake resistant masonry

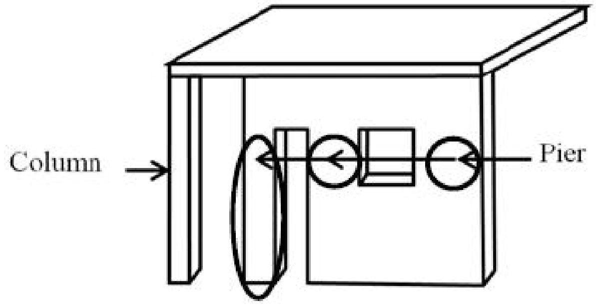
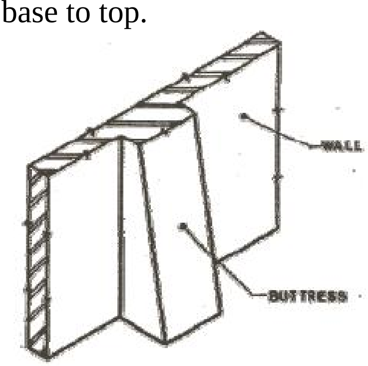
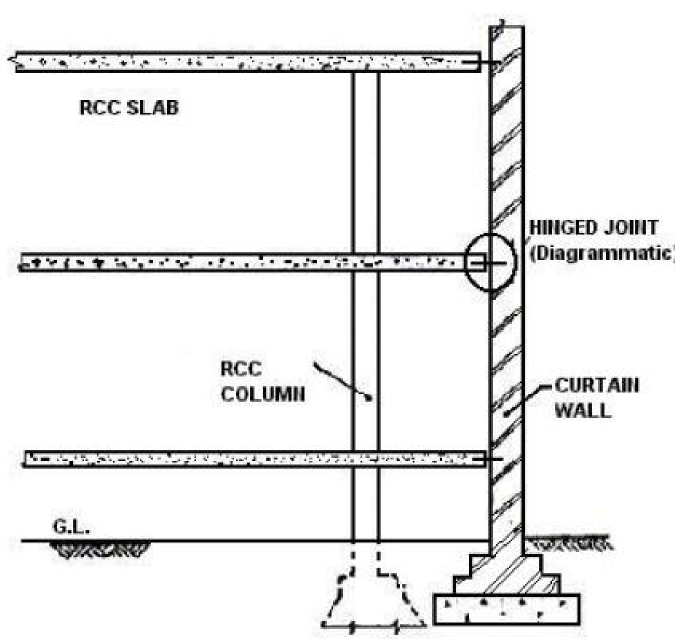
Terminologies in Structural Masonry

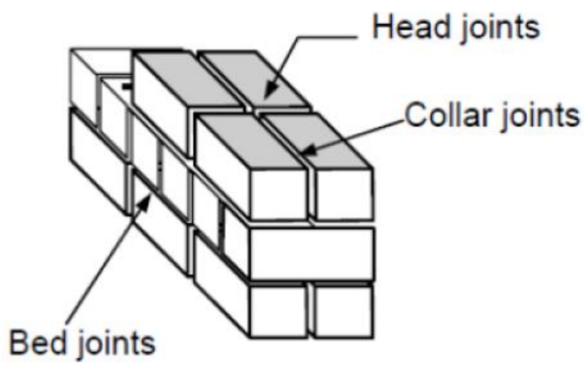
Table 1: Terminologies and abbreviations commonly referred in Structural Masonry

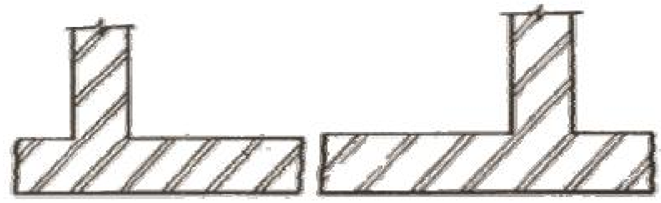
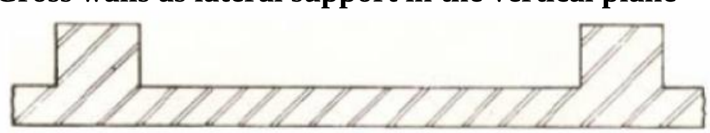
Sl. No.	**Terminology**	**Definition and remarks**
	Bed Block	A block bedded on a wall, column or pier to disperse a concentrated load on a masonry element.
	Cross-Sectional Area of Masonry Unit	Net cross-sectional area of a masonry unit shall be taken as the gross cross-sectional area minus the area of cellular space. Gross cross-sectional area of cored units shall be determined to the outside of the coring but cross-sectional area of grooves shall not be deducted from the gross cross-sectional area to obtain the net cross sectional area <u>Remark:</u> Net section area is difficult to ascertain especially in hollow masonry units. In case of full mortar bedding as shown in Fig 10.1 it is the gross sectional area based on the out-to-out dimension minus hollow spaces. Often alignment of cross webs is not possible while laying hollow units and the load transfer takes place through mortars on the face shells only. In such cases, it is conservative to base net cross-sectional area on the minimum face shell thickness.  Net cross sectional area = shaded area or gross area if the block is more than 75% solid  **Net area = shaded area = full-mortar bedding area** 

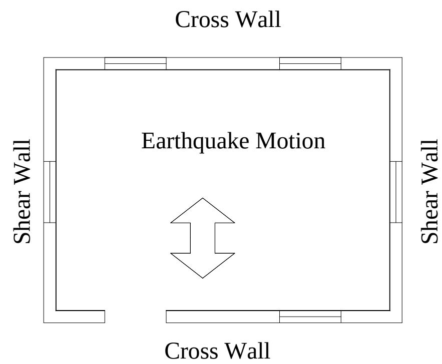
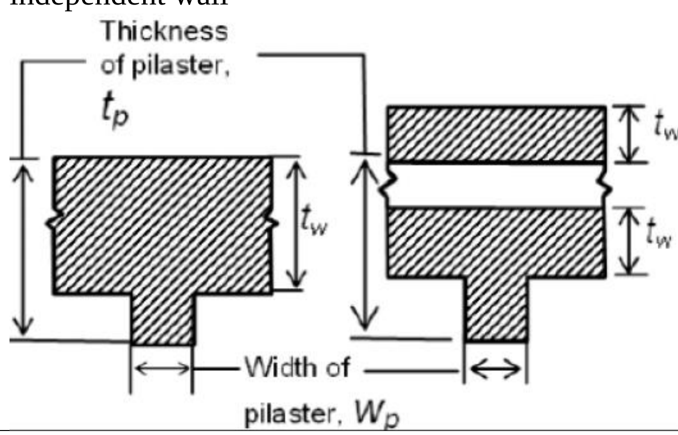
		Net area = shaded area if face-shell bedding is adopted (provided alignment of cross webs is ensured)
	Grout	A mixture of cement (or any binding material), sand and water of pourable consistency for filling small voids. Remark: used extensively for filling the surrounding the reinforcement in masonry
	URM	Un-reinforced masonry
	RM	Reinforced masonry
	MI	Masonry In-fill, the masonry wall between the columns and beams of a frame structure
	EMU	Engineered Masonry Unit – engineered for architectural (colour, shape, texture etc), physical (density) and structural requirement (strength, elasticity and durability)
	HCB	Hollow concrete block (A masonry unit of which net cross-sectional area in any plane parallel to the bearing surface is less than 75 percent of its gross cross-sectional area measured in the same plane)
	ECB	Engineered Concrete Block
	SMB	Stabilized Mud block
	SCB	Solid Concrete Block
	TMB	Table Moulded Brick
	WCB	Wire-cut Brick
	Grouted Hollow Masonry Unit	That form of grouted masonry construction in which certain designated cells of hollow units are continuously filled with grout.
	Grouted Multi-Wythe Masonry	That form of grouted masonry construction in which the space between the wythes is solidly or periodically filled with grout.
	Wythe	A continuous vertical tie of masonry one unit in thickness.
	Grouted Multi-Wythe Masonry	That form of grouted masonry construction in which the space between the wythes is solidly or periodically filled with grout.
	Joint Reinforcement	A prefabricated reinforcement in the form of lattice truss which has been hot dip galvanized after fabrication and is to be laid in the mortar bed joint.   **Ladder type reinforcement** **Truss type reinforcment**
	Prism	An assemblage of masonry units bonded by mortar with or without grout used as a test specimen for determining properties of masonry. (preferably with a height/thickness ratio between 2 to 5)
	Grouted Cavity	Two parallel single leaf walls spaced at least 50 mm apart, effectively

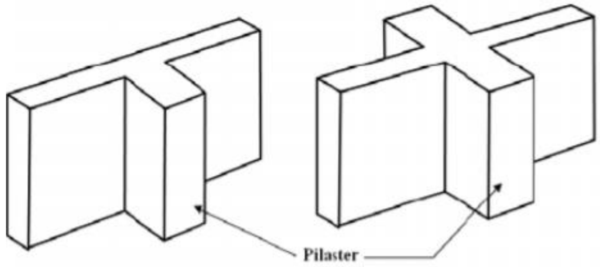
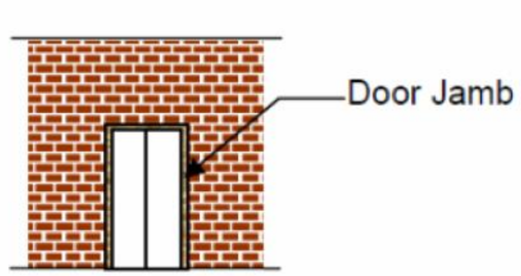
	Reinforced Masonry	tied together with wall ties. The intervening cavity contains steel reinforcement and is filled with infill concrete so as to result in common action with masonry under load.
	Pocket type Reinforced Masonry	Masonry reinforced primarily to resist lateral loading where the main reinforcement is concentrated in vertical pockets formed in the tension face of the masonry and is surrounded by in situ concrete.
	Quetta Bond Reinforced Masonry	Masonry at least one and half units thick in which vertical pockets containing reinforcement and mortar or concrete infill occur at intervals along its length.  **Quetta bond**
	Specified Compressive Strength of Masonry	Minimum Compressive strength, expressed as force per unit of net cross- section area, required of the masonry used in construction by the contract document, and upon the project design is based. Remark: Whenever the quantity f_m is under the radical sign, the square root of numerical value only is intended and the result has units of MPa.
	Wall Tie	A metal fastener which connects wythes of masonry to each other or to other materials.
	Bond	Arrangement of masonry units in successive courses to tie the masonry together both longitudinally and transversely; the arrangement is usually worked out to ensure that no vertical joint of one course is exactly over the one in the next course above or below it, and there is maximum possible amount of lap.
	Column	An isolated vertical load bearing member, width of which does not exceed four times the thickness.
	Pier	It is an isolated vertical member whose horizontal dimension measured at right angles to its thickness is not less than 4 times its thickness and whose height is less than 5 times its length.

		 **Column and pier**
	Buttress	A pilaster of masonry built as an integral part of wall and projecting from either or both surfaces, decreasing in cross-sectional area from base to top.  **Buttress**
	Curtain Wall	A non-load bearing wall subject to lateral loads. It may be laterally supported by vertical or horizontal structural members, where necessary  **Curtain wall**
	Effective Height	The height of a wall or column to be considered for calculating slenderness ratio.

	Effective Length	The length of a wall to be considered for calculating slenderness ratio.
	Effective Thickness	The thickness of a wall or column to be considered for calculating slenderness ratio.
	Joint	A junction of masonry units Remark: • Horizontal joints are known as bed joints • Vertical joints are known as perpends, and if they are perpendicular to the plane of the wall they are known as cross joint • Vertical joints are known as collar joints if they are parallel to the plane of the wall • Wall joints are the junctions of walls  **Joints**
	Leaf	Inner or outer section of a cavity wall.
	Lateral Support	A support which enables a masonry element to resist lateral load and/or restrains lateral deflection of a masonry element at the point of support. Remark: Lateral support is a primary requirement in structural design of masonry. A lateral support may be provided along either a horizontal or a vertical line, depending on whether the slenderness ratio is based on a vertical or horizontal dimension. Horizontal or vertical lateral supports should be capable of transmitting design lateral forces to the elements of construction that provide lateral stability to the structure as a whole.

		 **RC slab as a lateral support at the top of wall in the horizontal plane**  **Cross walls as lateral support in the vertical plane**  **Pilasters as lateral supports in the vertical plane**
	Load Bearing Wall	A wall designed to carry an imposed vertical load in addition to its own weight, together with any lateral load.
	Masonry Unit	Individual units which are bonded together with the help of mortar to form a masonry element, such as wall, column, pier and buttress.
	Partition Wall	An interior non-load bearing wall, one storey or part storey in height.
	Panel Wall	An exterior non-load bearing wall in framed construction, wholly supported at each storey but subjected to lateral loads in out-plane direction such as wind loads, earthquake loads etc.
	Shear Wall and Cross wall	A wall designed to carry horizontal forces acting in its plane with or without vertical imposed loads. The walls normal to shear walls are known as cross walls.

		 **Shear walls and cross walls**
	Slenderness Ratio (SR)	Ratio of effective height or effective length to effective thickness of a masonry element.
	Cavity Wall	A wall comprising two leaves, each leaf being built of masonry units and separated by a cavity and tied together with metal ties or bonding units to ensure that the two leaves act as one structural unit, the space between the leaves being either left as continuous cavity or filled with a non-load bearing insulating and waterproofing material.
	Faced Wall	A wall in which facing and backing of two different materials are bonded together to ensure common action under load backing shall be provided by toothing, bonding or other means.
	Veneered Wall	A wall in which the facing is attached to the backing but not so bonded as to result in a common action under load.
	K_s	Stress reduction factor
	K_a	Area reduction factor
	K_p	Shape modification factor
	Pilaster	A thickened section forming integral part of a wall placed at intervals along the wall, to increase the stiffness of the wall or to carry a vertical concentrated load. Thickness of a pilaster is the overall thickness including the thickness of the wall or when bonded into a leaf of a cavity wall, the thickness obtained by treating that leaf as an independent wall 

		 **Pilasters**
	Jamb	Side of an opening in wall.  **Jamb (example: door jamb)**
	Non-Load Bearing Wall	A wall that is not resisting or supporting any loads such that it can be removed with the approval of a structural engineer without jeopardizing integrity of the remaining structure
	Partition Wall	An interior non-load bearing wall, one storey or part storey in height.
	Veneered Wall	A wall in which the facing is attached to the backing but not so bonded as to result in a common action under load.
	Wall Tie	A metal fastener which connects wythes of masonry to each other or to other materials.

Masonry reinforcement

For the purpose of general load bearing construction, Fe 415 grade steel is acceptable, with the generic requirements as given in Table 2. However, for the purpose of earthquake resistant masonry, a variety of reinforcement can be used, typically the ones which impart to the system ductility.

Table 2: Specification for reinforcement in load bearing masonry

Tensile strength	
MS Bars confirming to IS 432 (Part I)	140 MPa for diameter ≤20 mm 130 MPa for diameter >20 mm
HYSD Bars (IS 1786)	230 MPa
Compressive strength	
MS Bars confirming to IS 432 (Part I)	130 MPa
Size and spacing of reinforcement	
The maximum size of reinforcement used in masonry shall be 25 mm diameter bars and minimum size shall not be less than 5 mm.	

The diameter of reinforcement shall not exceed one-half the least clear dimension of the cell, bond beam, or collar joint in which it is placed.
Clear distance between parallel bars shall not be less than the diameter of the bars, or less than 25 mm. In columns and pilasters, clear distance between vertical bars shall not be less than 1.5 times the bar diameter, nor less than 35 mm.

Basics of Load Bearing Masonry

It is very important to note that the first step in masonry building design is to ensure a stable configuration. Masonry structures gain stability from the support offered by cross walls, floors, roof and other elements such as piers and buttresses Load bearing walls are structurally more efficient when the load is uniformly distributed and the structure is so planned that eccentricity of loading on the members is as small as possible. Avoidance of eccentric loading by providing adequate bearing of floor/roof on the walls providing adequate stiffness in slabs and avoiding fixity at the supports etc., is especially important in load bearing walls in multistory structures. These matters should receive careful consideration during the planning stage of masonry structures.

In order to ensure uniformity of loading, openings in walls should not be too large. and these should be of 'hole in wall' type as far as possible; Bearings for lintels and bed blocks under beams should be liberal in sizes; heavy concentration of loads should be avoided by judicious planning and sections of load bearing members should be varied where feasible with the loadings so as to obtain more or less uniform stress in adjoining parts of members. One of the commonly occurring causes of cracks in masonry is wide variation in stress in masonry in adjoining parts.

Achieving lateral stability through lateral supports

Lateral support may be in the vertical or horizontal direction, the former consisting of floor/roof bearing on the wall ‘or properly anchored to the same and latter consisting of cross walls, piers or buttresses. These can be achieved by;

a) In case of a wall, where slenderness ratio is based on effective height, any of the following constructions are provided:

(i) RCC floor/roof slab (or beams and slab), irrespective of the direction of span, bears on the supported wall as well as cross walls to the extent of at least 9 cm;

(ii) RCC floor/roof slab not bearing on the supported wall or cross wall is anchored to it with non-corrodible metal ties of 60 cm length and of section not less than 6 x 30 mm, and at intervals not exceeding 2 m as shown in Fig. 1;

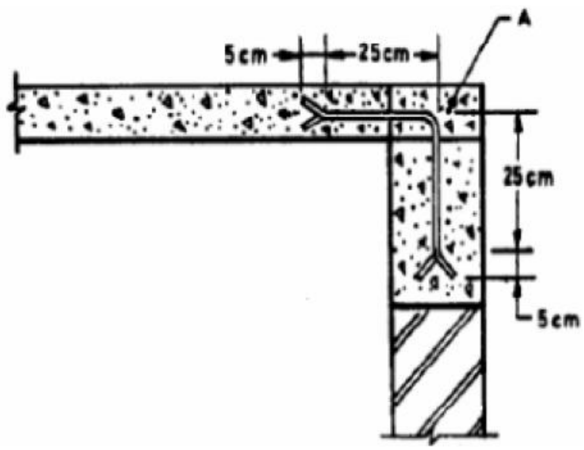


Fig 1: Anchoring a slab when it is not bearing on the wall

(iii) Timber floor/roof and pre-cast floor/roof require special connection details (not covered in this part)

In case of a wall, when slenderness ratio is based on its effective length; a cross wall/pier/buttress of thickness equal to or more than half the thickness of the supported wall or 90 mm, whichever is more, and length equal to or more than one-fifth of the height of wall is built at right angle to the wall (Fig 2) and bonded to it according to provision of 4.2.2.2 (d) of IS 1905 (1987)

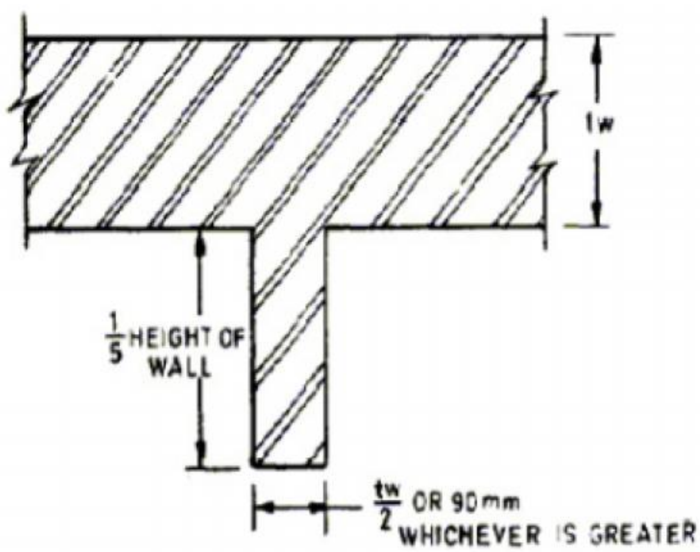


Fig 2: Minimum dimensions for masonry wall/buttress providing effective lateral support

b) In case of a column, an RCC or timber beam/R S joist/roof truss is supported on the column. In this case, the column will not be deemed to be laterally supported in the direction right angle to it; and

c) In case of a column, an RCC beam forming a part of beam and slab construction is supported on the column, and slab adequately bears on stiffening walls. This construction will provide lateral support to the column in the direction of both horizontal axes.

Achieving stability – general

A wall or column subjected to vertical and lateral loads may be considered to be provided with adequate lateral support from consideration of stability, if the construction providing the support is capable of resisting the following forces:

a) Simple static reactions at the point of lateral support to all the lateral loads; plus

b) 2.5 percent of the total vertical load that the wall or column is designed to carry at the point of lateral support.

In case of **load bearing un-reinforced buildings up to four storeys**, stability requirements of may be deemed to have been met with if:

a) Height to width ratio of building does not exceed 2;

b) Cross walls acting as stiffening walls continuous from outer wall to outer wall or outer wall to a load bearing inner wall, and of thickness and spacing as given in Table 10.7 are provided. If stiffening wall or walls that are in a line, are interrupted by openings, length of solid wall or walls in the zone of the wall that is to be stiffened shall be at least one-fifth of height of the opening as shown in Fig 10.8;

c) Floors and roof either bear on cross walls or are anchored to those walls as stated earlier, such that all lateral loads are safely transmitted to those walls and through them to the foundation;

d) And cross walls are built jointly with the bearing walls and are jointly mortared, or the two interconnected by toothing. Alternatively, cross walls may be anchored to walls to be supported by ties of non-corrodible metal of minimum section 6 x 35 mm and length 60 cm with ends bent up at least 5 cm; maximum vertical spacing of ties being 1.2 m).

Table 3: General guidelines for geometry of stiffeners

Thickness (m) of load bearing wall to be stiffened	Height (m) of storey <u>not</u> to exceed	Stiffening wall		
		Thickness (m) <u>not</u> less than		Maximum spacing (m)
		1 to 3 storey	4 storey	
0.1	3.2	0.1	-	4.5
0.2	3.2	0.1	0.2	6.0
0.3	3.4	0.1	0.2	6.0
Above 0.3	5.0	0.1	0.2	8.0

Remark

In case of halls exceeding 8.0 m in length, safety and adequacy of lateral supports shall always be checked by structural analysis.

Trussed roofing may not provide lateral support, unless special measures are adopted to brace and anchor the roofing. However, in case of residential and similar buildings of conventional design with trussed roofing having cross walls, it may be assumed that stability requirements are met with by the cross walls and structural analysis for stability may be dispensed with.

Capacity of a cross wall and shear wall to take horizontal loads and consequent bending moments, increases when parts of bearing walls act as flanges to the cross wall. Maximum overhanging length of bearing wall which could effectively function as a flange should be taken as *12 t* or *H/6*, whichever is less, in case of *T or I* shaped walls and *6 t* or *H/6*, whichever is less, in case of *L or U* shaped walls, where *t* is the thickness of bearing wall and *H* is the total height of wall above the level being considered.

The connection of intersecting walls shall conform to one of the following requirements:

c) Providing proper masonry bonds such that 50% of masonry units at the interface shall interlock.

b) Connector or reinforcement extending in each of the intersecting wall shall have strength equal to that of the bonded wall

c) Requirements of section 8.2.4 of IS: 4326.

Effective overhanging width of flange = 12 *t* or *H*/6 whichever is less, *H* being the total height of wall above the level being considered. Effective overhanging width of flange = 6 *t* or *H*/6 whichever is less, *H* being the total height of wall above the level being considered In case of external walls of basement and plinth stability requirements may be deemed to have been met with if:

a) bricks used in basement and plinth have a minimum crushing strength of 5 MPa and mortar used in masonry is of Grade Ml or better;
b) clear height of ceiling in basement does not exceed 2.6 m;
c) walls are stiffened according to provisions of 4.2.2.1;
d) in the zone of action of soil pressure on basement walls, traffic load excluding any surcharge due to adjoining buildings does not exceed 5 kN/m^2 and terrain does not rise; and
e) Minimum thickness of basement walls is in accordance with Table 4. In case there is surcharge on basement walls from adjoining buildings, thickness of basement walls shall be based on structural analysis.

Table 4: Minimum thickness of basement walls

Height of the ground above basement floor level with wall loading (permanent load)		Minimum thickness (m) of basement walls
More than 50 kN/m	Less than 50 kN/m	
2.75	2.0	0.4
1.75	1.4	0.3

Structural design

The building as a whole shall be analyzed by accepted principles of mechanics to ensure safe and proper functioning in service of its component parts in relation to the whole building. All component parts of the structure shall be capable of sustaining the most adverse combinations of loads, which the building may be reasonably expected to be subjected to during and after construction.

Some general guidance on the design concept of load bearing masonry structures is given in the following paragraphs.

A building is basically subjected to two types of loads, namely:

1. vertical loads on account of dead loads of materials used in construction, plus live loads due to occupancy; and
2. lateral loads due to wind and seismic forces.

While all walls in general can take vertical loads, ability of a wall to take lateral loads depends on its disposition in relation to the direction of lateral load. The lateral loads acting on the face of a building are transmitted through floors (which act as horizontal beams) to cross walls which act as shear walls. From cross walls, loads are transmitted to the foundation. This action is illustrated in Fig. 3. Wind load on the facade wall is transferred via floor slabs to the cross walls and thence to the ground. The strength and stiffness of floors as horizontal girders is vital; hence floors/roofs of lightweight construction should be used with care.

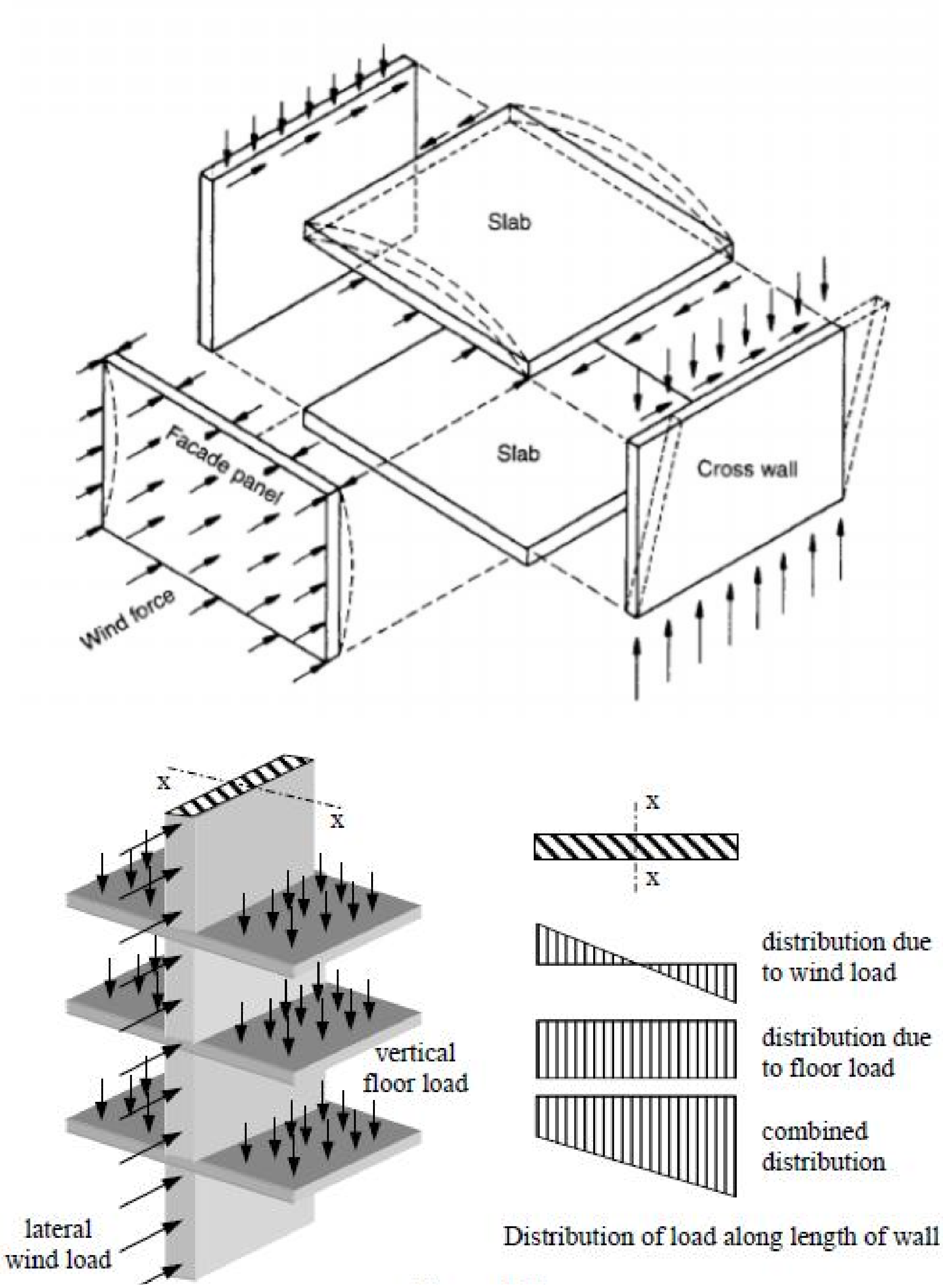


Fig 3: Lateral force (eg. wind force) is resisted by the facade panel owing to bending, and transferred via floor slabs to the cross or shear wall and finally to the ground.

As a result of lateral load, in the cross walls there will be an increase of compressive stress on the leeward side, and decrease of compressive stress on the wind-ward side. These walls should be designed for '**no tension**' and **permissible compressive stress**. It will be of interest to note that a wall which is carrying greater vertical loads will be in a better position to resist lateral loads than the one which is lightly loaded in the vertical direction. This point should be kept in view while planning the structure so as to achieve economy in structural design.

A structure should have adequate stability in the direction of both the principal axes. The so-called 'cross wall' construction may not have much lateral resistance in the longitudinal direction. In multi-storeyed buildings, it is desirable to adopt 'cellular' or 'box type' construction from consideration of stability and economy.

Size, shape and location of openings in the external walls have considerable influence on stability and magnitude of stresses due to lateral loads.
If openings in longitudinal walls are so located that portions of these walls act as flanges to cross walls, the strength of the cross walls get considerably increased and structure becomes much more stable.

Ordinarily a load-bearing masonry structure is designed for permissible compressive and shear stresses (with no tension) as a vertical cantilever by accepted principles of engineering mechanics. No moment transfer is allowed for, at floor to wall connections and lateral forces are assumed to be resisted by diaphragm action of floor/roof slabs, which acting as horizontal beams, transmit lateral forces to cross walls in proportion to their relative (moment of inertia).

Design Loads

Loads to be taken into consideration for designing masonry components of a structure are:

a. dead loads of walls, columns, floors and roofs;
b. live loads of floors and roof;
c. wind loads on walls and sloping roofs and
d. seismic forces.

Note - When a building is subjected to other loads, such as vibration from railways and machinery, these should be taken into consideration according to the best engineering judgment of the designer.

Dead loads

Dead loads shall be calculated on the basis of unit weights taken in accordance with IS:875 – part I (1987).

Live Loads and Wind Loads

Design loads shall be in accordance with the recommendations of IS: 875- (1987) or such other loads and forces as may reasonably be expected to be imposed on the structure either during or after construction.

Note - During construction, suitable measures shall be taken to ensure that masonry is not liable to damage or failure due to action of wind forces, back filling behind walls or temporary construction loads.

Seismic loads

Seismic loads shall be determined in accordance with the IS 1893- Part 1:2002.

Load combinations

In the allowable stress design method followed for the structural design of masonry structures as outlined in this code, adequacy of the structure and member shall be investigated for the following load combinations:

a) DL + IL
b) DL + IL + (WL or EL)
c) DL + WL
d) 0.9 DL +EL

Note: The four load combinations given are consistent with those in other BIS codes. In case of wind and earthquake loads, the reversal of forces needs to be considered. The structure is to be designed for the critical stresses resulting from these load combinations.

Permissible stresses and loads

Permissible stresses and loads may be increased by one-third for load case b, c, & d when wind or earthquake loads are considered along with normal loads.

As an alternative of using an increased permissible stress value when checking safety of structural components, one can use a 25% reduced load for load combinations involving wind or earthquake forces and compare with full permissible stress values. Thus, the modified load combinations b, c and d will be:

a) 0.75 [DL + IL + (WL or EL)]
b) 0.75 [DL + WL]
c) 0.75 [0.9DL +EL]

Vertical load dispersion

Generally, it is accepted, based on experiments, that dispersion of axial loads does not take place at an angle 45° to vertical as assumed in previous codes. An angle of distribution for axial loads not exceeding 30° is more realistic and is recommended by various other masonry codes.

In case of buildings of conventional design with openings of moderate size which are reasonably concentric, some authorities on masonry recommend a simplified approach for design. In simplified approach, stress in masonry at plinth level is assumed to be uniformly distributed in different stretches of masonry, taking loadings in each stretch of masonry walls without making any deduction in weight of masonry for the openings. **It is assumed that the extra stresses obtained in masonry by making no deduction for openings, compensates more or less for concentrations of stresses due to openings**. This approach is of special significance in the design of multi-storeyed load-bearing structure where intervening floor slabs tend to disperse the upper storey loads more or less uniformly on the inter-opening spaces below the slabs and thus at plinth level stress in masonry, as worked out by the above approach is expected to be reasonably accurate.

Lintels

Lintels, that support masonry construction, shall be designed to carry loads for masonry (allowing for arching and dispersion, where applicable) and loads received from any other part of the structure. Length of bearing of lintel at each end shall not be less than 9 cm or one-tenth of the span, whichever is more, and area of the bearing shall be sufficient to ensure that stresses in the masonry (combination of wall stresses, stresses due to arching action and bearing stresses from the lintel) do not exceed the stresses permitted.

When location and size of opening is such that arching action can take place, lintel is designed for the load of masonry included in the equilateral triangle over the lintel. In case floor or roof slab falls within a part of the triangle in question or the triangle is within the influence of a concentrated load or some other opening occurs within a part of the triangle, loading on the lintel will get modified as discussed earlier.

Lateral load distribution

Lateral loads from the wind or earthquakes are generally considered to act in the direction of the principal axes of the building structure. The distribution of lateral loads to various masonry wall elements depends on the rigidities of the horizontal floor or roof diaphragm and of the wall elements. If a diaphragm does not undergo significant in-plane deformation with respect to the supporting walls, it can be considered rigid and lateral loads are distributed in various lateral load resisting wall elements in proportion to their relative stiffness. Horizontal torsion developed due to eccentricity of the applied lateral load with the plan centre of the rigidity can cause forces in the wall parallel and perpendicular to load direction. In-plane rigidities are considered in the analysis, which includes both shearing and flexural deformations. Generally rigidities of transverse walls in direction perpendicular to the direction of lateral force, is usually disregarded. However, stiffening effect of certain portion of such walls as permitted by the code, when the stiffening action is significant, i.e. when the method of connection between the intersecting walls and between walls and diaphragms is adequate for the expected load transfer. On the other hand, flexible diaphragms change shape when subjected to lateral loads and are incapable of transmitting torsional forces. The distribution of lateral loads to vertical wall elements takes place in proportion to the tributary area associated with each wall element for vertical loads distribution.

Basic Compressive Strength of Masonry

The basic compressive strength of masonry f_m shall be determined by the (a) unit strength method or by the (b) prism test method. The unit strength method eliminates the expense of prism tests but is more conservative than the prism test method.

(a) Unit strength method

The basic compressive strength of masonry shall be four times of the basic compressive stress which based on the strength of the units and the type of mortar. Unit strength method is based on the compressive strength of masonry units and mortar type, and is developed by using prism test data.

(b) Prism strength method

Basic compressive strength of masonry shall be determined by prism test on masonry made from masonry units and mortar to be actually used in a particular job. This is a uniform method of testing masonry to determine its compressive strength and is used as an alternative to the unit strength method.

Permissible stresses

Permissible compressive stress in masonry shall be based on the value of basic compressive stress (f_b) which is based on two approaches, (i) when prism is not tested and (ii) when prism is tested.

Prism not tested/Unit Strength Method:

Values of basic compressive stress given in Table 5 which are based on the crushing strength of masonry unit and grades of mortar, and hold good for values of SR not exceeding 6, zero eccentricity and masonry unit having height to width ratio (as laid) equal to 0.75 or less.

Prisms tested:

The basic compressive stress can be obtained by multiplying the specified compressive strength obtained from prism test with a factor of 0.25.

Permissible Compressive Stress

Permissible compressive stress in masonry shall be based on the value of basic compressive stress (f_b) as given in Table 4 and multiplying this value by factor known as stress reduction factor (k_s), Area reduction factor (k_a) and shape modification factor (k_p). Amongst these, the stress reduction factor plays a very important role. It can be explained with the help of fig. 4 and to fig. 5. When the prism (or a short wall) is axially loaded, it can withstand maximum load. As the wall becomes slender, the load carrying capacity reduces and when the loads are eccentric, the load carrying capacity becomes even lesser. Thus the slenderness ratio (SR) and the eccentricity of load (or e/t ratio) plays an important role is the estimation of load capacity of walls. This is presented in Table 6. In the present Indian code, the stress reduction factors are unity for SR=6 and all values of e/t, this is not the case in the other masonry codes. Also the stress reduction factors are to be taken for any type of masonry, but current literature indicates clearly that both, the strength and elasticity of masonry play a role in the reduction factors.

Area reduction factor due to 'small area' of a member is based on the concept that there is statistically greater probability of failure of a small section due to sub-standard units as compared to a large element. However some codes do not include any provision for smallness of area. In view of the fact that strength of masonry units being manufactured at present in our country can appreciably vary, the necessity for this provision is justified in our code. This factor is applicable when sectional area of the element is less than 0.2 m^2. The factor k_a=0.7 + 1.5 A, A being the area of section in m^2.

Shape modification factor is based on the general principle that lesser the number of horizontal joints in masonry, greater its strength or load carrying capacity. This is presented in table 5. Here also there is a need for further studies.

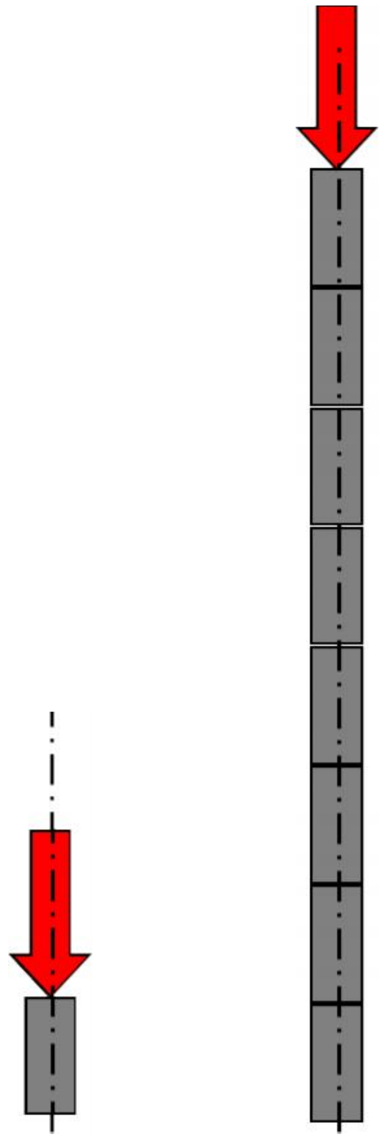


Fig. 4: (a) Short and axially loaded wall (capacity 100%) (b) Slender and axially loaded wall (capacity < 100%)

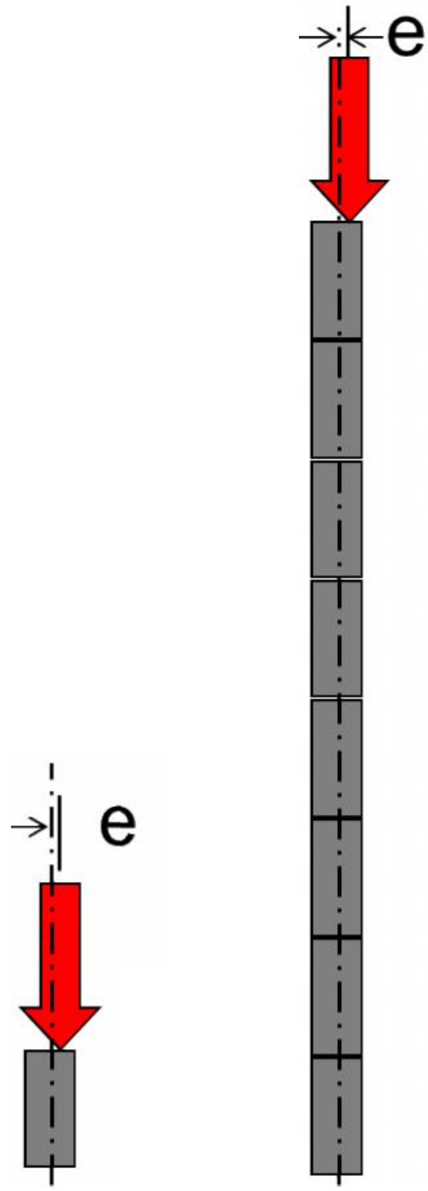


Fig. 5: (a) Short and eccentrically loaded wall (capacity < 100%) (b) Slender and eccentrically loaded wall (capacity << 100%)

Table 5: Basic Compressive strength (in MPa)

Sl. no	Mortar Type	Table 10: Basic compressive strength in MPa corresponding to masonry units of which height to width ratio does not exceed 0.75 and crushing strength in MPa is not less than											
		3.5	**5.0**	**7.5**	**10**	**12.5**	**15**	**17.5**	**20**	**25**	**30**	**35**	**40**
1	**H1**	0.35	0.50	0.75	1.00	1.16	1.31	1.45	1.59	1.91	2.21	2.50	3.05
2	**H2**	0.35	0.50	0.74	0.96	1.09	1.19	1.30	1.41	1.62	1.85	2.10	2.50
3	**M1**	0.35	0.50	0.74	0.96	1.06	1.13	1.20	1.27	1.47	1.69	1.90	2.20
4	**M2**	0.35	0.44	0.59	0.81	0.94	1.03	1.10	1.17	1.34	1.51	1.65	1.90
5	**M3**	0.25	0.41	0.56	0.75	0.87	0.95	1.02	1.10	1.25	1.41	1.55	1.78
6	**L1**	0.25	0.36	0.53	0.67	0.76	0.83	0.90	0.97	1.11	1.26	1.40	1.06
7	**L2**	0.25	0.31	0.42	0.53	0.58	0.61	0.65	0.69	0.73	0.78	0.85	0.95

Table 6: Stress reduction factors (k_s)

Slenderness Ratio	Eccentricity of loading divided by the thickness of the member					
	0	1/24	1/12	1/6	1/4	1/3
(1)	(2)	(3)	(4)	(5)	(6)	(7)
6	1.00	1.00	1.00	1.00	1.00	1.00
8	0.95	0.95	0.94	0.93	0.92	0.91
10	0.89	0.88	0.87	0.85	0.83	0.81
12	0.84	0.83	0.81	0.78	0.75	0.72
14	0.78	0.76	0.74	0.70	0.66	0.66
16	0.73	0.71	0.68	0.63	0.58	0.53
18	0.67	0.64	0.61	0.55	0.49	0.43
20	0.62	0.59	0.55	0.48	0.41	0.34
22	0.56	0.52	0.48	0.40	0.32	0.24
24	0.51	0.47	0.42	0.33	0.24	-
26	0.45	0.40	0.35	0.25	-	-
27	0.43	0.38	0.33	0.22	-	-

Table. 7: Shape modification factor

Height to width ratio of units(as laid)	Shape modification factor (k_p) for units having crushing strength in MPa			
	5.0	7.5	10.0	15.0
(1)	(2)	(3)	(4)	(5)
Up to 0.75	1.0	1.0	1.0	1.0
1.0	1.2	1.1	1.1	1.0
1.5	1.5	1.3	1.2	1.1
2.0 to 4.0	1.8	1.5	1.3	1.2

Combined Permissible Axial and Flexural Compressive Stress

Members subjected to combined axial compression and flexure shall be designed to satisfy the following:

$$\frac{f_A}{F_a} + \frac{f_B}{F_b} \leq 1$$

Where,

f_a= Calculated compressive stresses due to axial load only

f_b= Calculated Compressive stresses due to flexure only

F_a = Allowable axial compressive stress

F_b = Allowable flexural compressive stress = 1.25 F_a

The unity equation assumes a straight line interaction between axial and flexural compressive stresses for unreinforced masonry sections. This is simple portioning of the available allowable stresses between axial and flexure loads, which can be extended for the biaxial bending, by using the bending stress quotients for both axes. In this interaction formula, the secondary effect of moment magnification for flexure term due to axial loads is not included, which is an error on the unsafe side. However, this error for practical size of walls will be relatively small and large overall safety factor of about 4 is adequate to account for this amplification of flexure term. The code allows 25% increase in allowable axial compressive stress, if it is due to flexure. The permissible flexural compressive stress can be expressed as a function of masonry prism strength as follows:

$F_b = 1.25\ F_a = 1.25 \times 0.25\ f_m = 0.31\ f_m$

Permissible Tensile Stress

As a general rule, design of masonry shall be based on the assumption that masonry is not capable of taking any tension. However, in case of lateral loads normal to the plane of the wall, which causes flexural tensile stress, as for example, panel, .curtain partition and freestanding walls, flexural tensile stresses as follows may be permitted in the design for masonry:

Grade M1 or Better mortar

- 0.07 MPa for bending in the vertical direction where tension developed is normal to bed joints.
- 0.14 MPa for bending in the longitudinal direction where tension developed is parallel to bed joints provided crushing strength of masonry units is not less than 10 MPa.

Grade M2 mortar

- 0.05 MPa for bending in the vertical direction where tension developed is normal to bed joints.
- 0.10 MPa for bending in the longitudinal direction where tension developed is parallel to bed joints provided crushing strength of masonry units is not less than 7.5 MPa.

Important note:

No tensile stress is permitted in masonry in case of water-retaining structures in view of water in contact with masonry. Also no tensile stress is permitted in earth-retaining structures in view of the possibility of presence of water at the back of such walls.

Permissible shear stress

In-plane permissible shear stress (Fv)shall not exceed any of :

(a) 0.5 MPa

(b) $0.1 + 0.2 f_d$

(c) $0.125\ (f_m)^{1/2}$

Where,

f_d = compressive stress due to dead loads in MPa.

Unreinforced masonry in shear fails in one of the following mode (a) Diagonal tension cracking of masonry generally observed when masonry is weak and mortar is strong, (b) Sliding of masonry units along horizontal bed joint, especially when masonry is lightly loaded in vertical direction and (c) Stepped cracks running through alternate head and bed joints, usually observed in case of strong units and weak mortars. Permissible shear stress for unreinforced masonry is based on experimental research for various failure modes. At low pre-compression (<2 MPa), for sliding type of failure mode, a Mohr-Coulomb type failure theory is more appropriate and shear capacity is increased due to increase in the vertical load. The coefficient of friction of 0.2 has been long used in the masonry codes, however, the recent research indicate that a higher value (about 0.45) is more appropriate. At large pre-compression (> 2 MPa), tensile cracking of masonry is more likely which are expressed in terms of square root of compressive strength of masonry.

Wall Thickness (Cross-Section and Dimensions)

Walls and Columns Subjected to Vertical Loads: Walls and columns bearing vertical loads shall be designed on the basis of permissible compressive stress. Design involves in determining thickness in case of walls and the section in case of columns in relation to strength of masonry units and grade of mortar to be used, taking into consideration various factors such as slenderness ratio, eccentricity, area of section, workmanship, quality of supervision, etc.

Solid Walls

Thickness used for design calculation shall be the actual thickness of masonry computed as the sum of the average dimensions of the masonry units specified in the relevant standard, together with the specified joint thickness. In masonry with raked joints, thickness shall be reduced by the, depth of raking of joints for plastering/pointing. Brick work is generally finished by either pointing or plastering and with that in view, it is necessary to rake the joints while the mortar is green, in case of plaster work raking is intended to provide key for bonding the plaster with the background. Strictly speaking, thickness of masonry for purposes of design in these cases is the actual thickness less depth of raking. However in case of design of masonry based on permissible tensile stress (as for example, design of a free standing wall), if walls are plastered over (plaster of normal thickness i.e. 12 to 15 mm) with mortar of same grade as used in the masonry or M2 grade whichever is stronger or if walls are flush pointed with mortar of M1 grade or stronger, raking thickness can be ignored.

Concepts for earthquake resistant masonry

The basic principles of design and detailing, as outlined in the codes of practice, of earthquakes resistant structures are intentionally simple and generally easy to adopt. Essentially the principles are focused on,

(i) Achieving strength and ductile behaviour
(ii) Maintaining structural integrity

This means that the primary requirement is ‘prevention of catastrophic collapse of buildings or their components’. It is also the intention of the codes of practice to achieve this in relatively simple and cost effective manner.

The level of resistance aimed for in earthquake resistant design is based on the concept of ‘acceptable risk’, with the following objectives;

- To resist minor earthquakes without damage
- To resist moderate earthquakes without significant structural damage, but with some non-structural damage
- To resist major (or severe) earthquake without major failure of the structural framework of the building or its components, to prevent loss of life and to allow safe escape passage for the inmates of the building.

However, certain important critical structures hospitals, power generating units, communication set-ups etc., shall be designed to remain operational during and after an earthquake event.

Un-reinforced masonry buildings are very common in rural and semi-urban area of India. A variety of load bearing masonry units such as adobe, stone, burnt brick, concrete blocks and stabilized mud blocks are commonly used along with a variety of mortars such as mud mortar, cement mortar, lime mortar and composite mortar. Normally these buildings are designed for vertical loads and since masonry has adequate compressive strength, the structure behaves well as long as the loads are vertical.

The behaviour of a masonry building during ground motion can be understood by analysing the nature of stress distribution in the walls of the masonry building. When dominant ground motion is along one axis of the building, the walls parallel to the direction of ground motion are known as ‘shear walls’ and those orthogonal to it are known as ‘cross walls’.

Shear walls are predominantly subjected to in-plane shear stresses and in-plane bending stresses. The in-plane bending stresses in shear walls are normal-to-bed joints. The in-plane shear stresses are responsible for the typical X-type of cracking in the shear walls, while the in-plane bending stresses in the shear walls tend to cause separation of cross walls and shear walls at the junction. Although severe cracking could be caused, the walls may not readily collapse unless a component of ground motion is normal to it. The stress concentration near the openings in shear walls adds to the vulnerability.

The failure pattern of such masonry structures during earthquake can be classified as under (shown in plates 1 to 7);

a) Out-of-plane flexural and/or out-of-plane shear failure

b) In-plane shear and/or in-plane flexure failure

c) Separation of walls at junction

d) Failure of masonry piers between openings

e) Local failures

f) Buckling of wythes

g) Separation of roof from walls

Plate 1: Out-of-plane flexure failure

Plate 10.6: In-plane shear failure

Plate 2: Separation of wall at junctions

Plate 3: Failure of masonry piers between openings

Plate 4: Local failures

Plate 5: Buckling of wythes

Plate 6: Separation of roof from walls

Concept of 'Containment Reinforcement'

The pattern of failure of masonry buildings during an earthquake makes it clear that the prevention of sudden flexural failure of masonry wall is critical to ensure an earthquake resistant masonry structure. Since flexural tension can occur on both the faces of the wall due to reversal of stresses during an earthquake, there is a need to provide ductile reinforcement on both the faces. This can be accomplished by placing vertical reinforcement either on the surface or close to the surface and surrounding the wall, which is termed as "containment reinforcement". For the containment reinforcement to be effective, it is essential for it to remain hugged to the wall all times during an earthquake. In order to meet this objective and to prevent buckling of the reinforcement on the compression side of the wall, the vertical reinforcement on either face of the wall to be connected to each other, through horizontal ties/links passing through the bed joint of masonry. Containment reinforcement is intended to permit large ductile deformation and avoid total collapse. In other words, containment reinforcement will act as main energy absorbing element of the wall which otherwise is poor energy absorbing capacity. Fig 6 shows a schematic diagram of containment reinforcement for a typical masonry wall with ties at bed joints. The complete scheme of vertical and horizontal reinforcement is shown in Fig 7.

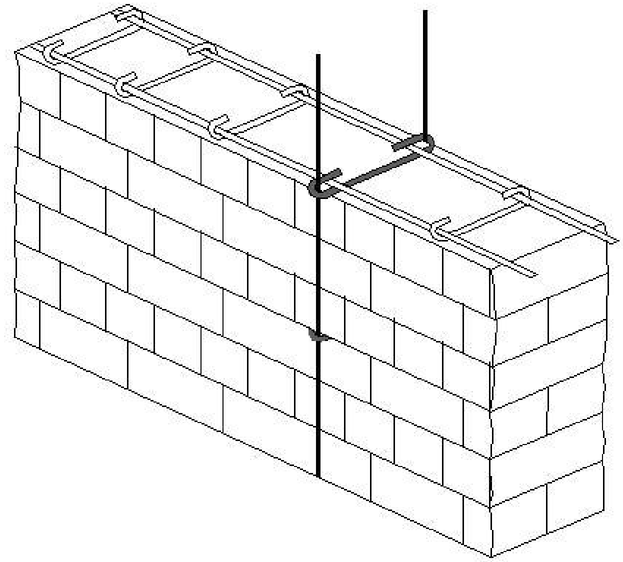


Fig 6: Containment reinforcement scheme integrated with horizontal bed reinforcement

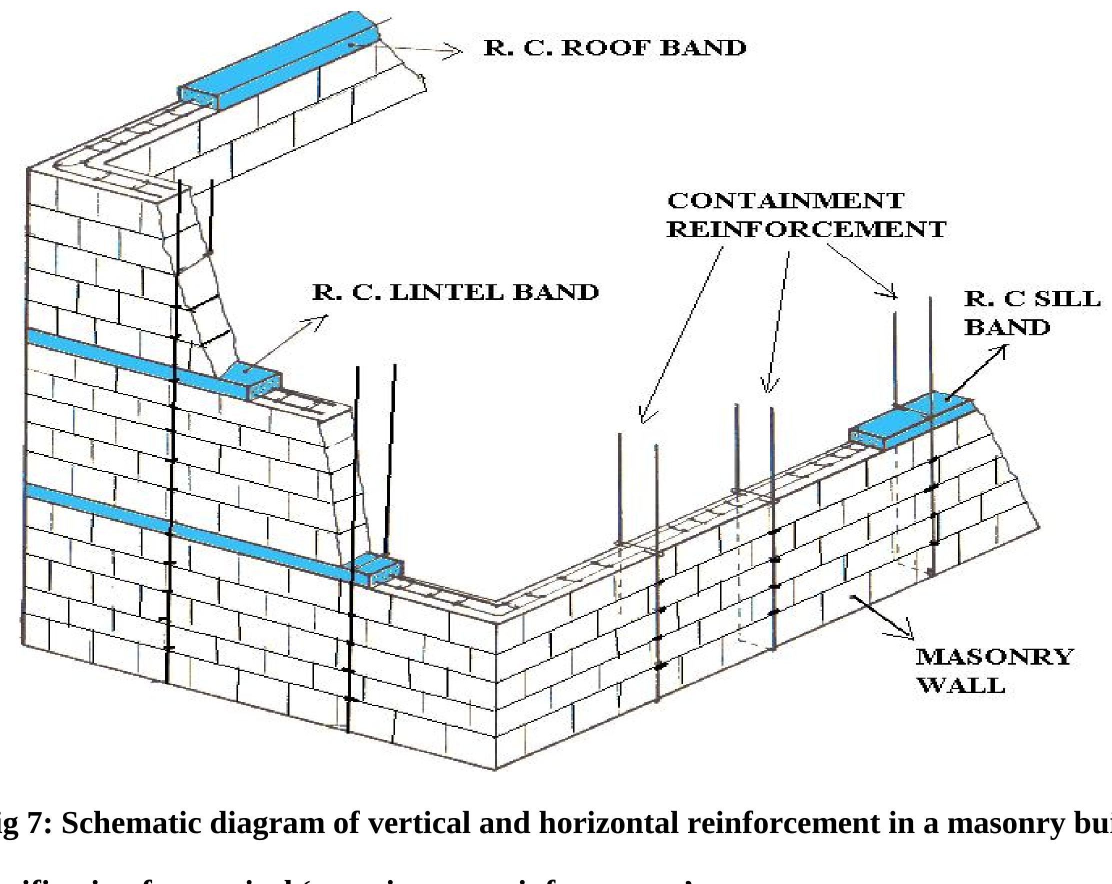


Fig 7: Schematic diagram of vertical and horizontal reinforcement in a masonry building

Specification for vertical 'containment reinforcement'

(i) It is recommended that containment reinforcement may be provided for low-rise (up to 3 storey load bearing) masonry buildings in earthquake zones III, IV, and V. This is in addition to horizontal bands.

(ii) In case of buildings with heavy roofs/floors (mass of the floor more than 200kg/m^2), if height of the wall is 3.0m or less and the length of the wall is less than or equal to 3.0m containment reinforcement need not be provided if there are no openings in the wall.

(iii) Masonry buildings with light roofs (tiled roof, asbestos or zinc sheet roofs) must have containment reinforcement on all walls irrespective of the aspect ratio of the wall.

(iv) Walls with height greater than 3.0m must invariably have containment reinforcement.

(v) All door and window jambs must have containment reinforcement on either sides of the opening at a distance of 150.0mm to 200.0mm from the jamb. Masonry piers between door and window openings or between two window openings should not be less than 0.75m in width. This is a modification of clause 8.3.1 in IS: 4326 (1993). Other provision in this clause may not be changed.

(vi) The wires/rods of containment reinforcement must be tied to the steel in the horizontal band to form a coarse two-dimensional cage holding the masonry in place.

(vii) Normally, the horizontal spacing between two sets of containment reinforcement should be between 0.75m to 1.25m.

(viii) A variety of reinforcing materials can be used as containment reinforcement. The details are presented in Table 8.

Table – 8: Different materials for 'containment reinforcement'

Reinforcing material	Remarks
Mild steel rods/flats	6mm rods available, very ductile, liable to corrosion if exposed and hence has to be either coated with non-corrosive paints or covered with plaster. 20-25mm wide, 3mm thick MS flats could also be used, holes could be made at regular intervals to insert links/bolts to tie the flats provided on both faces of the wall.
Galvanized Iron (GI) wires/flats	Any diameter wire available, easy for handling, good ductility, liable to corrosion and hence has to be protected. 20-25mm wide, 3mm thick GI flats could be used as mentioned above.
Stainless steel	Ideal material for containment reinforcement, 3mm to 4mm wires at 1.0m spacing, no need of coating, plastering etc.
Timber battens	Good quality battens (teak wood, sal wood etc.) of size 50mm x 25mm at 1.0m spacing, the pair of batten on either face of the wall to be tied together at two points at the base and two points at the top by boring a hole and inserting a bolt; needs regular maintenance to prevent rotting; care to be taken to prevent it from catching fire.
Bamboo/split bamboo	Pairs of bamboo or split (half) bamboos at about 1.0m to 1.5m interval; the poles to be tied at two points at the base and two points at top by using GI wires; less life; can catch fire, hence has to be protected
Ferro-cement strips	Thin ferro-cement strips (about 150.0mm wide) with sufficient amount

	of reinforcing material such as chicken mesh, expanded metal, weld mesh etc. at 1.2m spacing; the strips have to be bonded to the masonry wall by using grouted hooks.
Aluminum	Wires, rods and flats readily available, durable and have good resistance to corrosion, strength and modulus is less and hence large quantity is needed.

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Earthquake resistant design of structures (Subject Code: 06CV834)

UNIT 5 & Unit 6: Seismic lateral force analysis

Contents:

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1 Introduction:

Apart from gravity loads, the structure will experience dominant lateral forces of considerable magnitude during earthquake shaking. It is essential to estimate and specify these lateral forces on the structure in order to design the structure to resist an earthquake. It is impossible to exactly determine the earthquake induced lateral forces that are expected to act on the structure during its lifetime. However, considering the consequential effects of earthquake due to eventual failure of the structure, it is important to estimate these forces in a rational and realistic manner.

The earthquake forces in a structure depend on a number of factors such as,

- Characteristics of the earthquake (Magnitude, intensity, duration, frequency, etc.)
- Distance from the fault
- Site geology
- Type of structure and its lateral load resisting system.

2 Earthquake Resistant Design Philosophy

Apart from the factors mentioned above, the consequences of failure of the structure may also be of concern in the reliable estimation of design lateral forces. Hence, it is important to include these factors in the lateral force estimation procedures.

Code of practice for earthquake resistant design of structures primarily aims at accomplishing two primary objectives; total safety against loss of life and minimization of economic loss. These objectives are fulfilled by design philosophy with following criteria,

- Resist minor earthquake shaking without damage

- Resist moderate earthquake shaking without structural damage but possibly with some damage to nonstructural members
- Resist major levels of earthquake shaking with both structural and nonstructural damage, but the building should not collapse thus endangerment of the lives of occupants is avoided.

Conceptual representation of the earthquake resistant design philosophy is depicted in Figure 1.

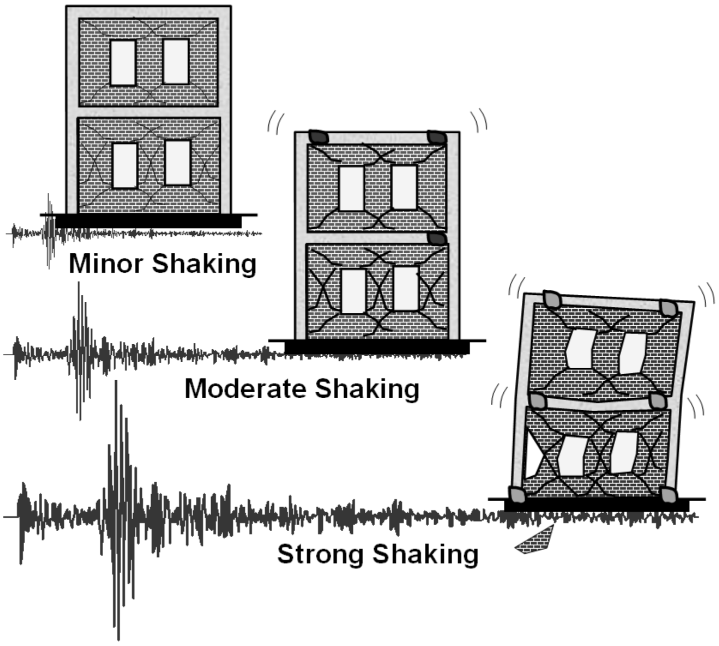

Figure 1: Schematic diagram depicting earthquake resistant design philosophy for different levels shaking [IITK-BMTPC (2004)]

The purpose of an earthquake-resistant design is to provide a structure with features, which will enable it to respond satisfactorily to seismic effects. These features are related to five major objectives, which are listed in order of importance:

- The likelihood of collapse after a very severe earthquake should be as low as possible.
- Damage to non-structural elements caused by moderate earthquakes should be kept within reasonable limits. Although substantial damage due to severe earthquakes, which have a low probability of occurrence is acceptable, such damage is unacceptable in the case of moderate tremors which are more likely to occur.

- Buildings in which many people are usually present should have deformability features which will enable occupants to remain calm even in the event of strong shocks.
- Personal injury should be avoided.
- Damage to neighboring buildings should be avoided

3 Guidelines for Earthquake Resistant Design

As mentioned above, the philosophy of earthquake design is to prevent non-structural damage in frequent minor ground shaking, is to prevent structural damage and minimize non-structural damage in occasional moderate ground shaking and to avoid collapse or serious damage in rare major ground shaking. In order to meet these requirements the code of practice for earthquake resistant design of structures generally prescribes guidelines with respect to following aspects,

- Intensity of shaking is prescribed based on zone factor depending upon seismic activity in the region of geographical location of the site
- Characteristics of the structures that affect its dynamic behaviour is accounted by prescribing appropriate natural period depending on distribution of mass and stiffness properties also, by considering type of soil beneath its foundation.
- Importance factor is assigned depending on occupancy type, functionality etc. of the structure
- Capability of a particular structure to resist lateral forces is incorporated by identifying its redundancy and ductility features through response modification factor.

When inertia of the structure offers resistance to ground motions, structure will experience earthquake forces. The relative movement between the ground and the structure induces a force dependent on the ground acceleration, mass and stiffness properties of the structure. The ground acceleration depends on the magnitude and intensity of the seismic event at a location. Based on seismic records, experience, and research, some areas of the country are determined to have a greater probability of earthquakes than others, and some areas have more severe earthquakes. This is taken into account by dividing the country into different zones that represent estimates of future earthquake occurrence and strength.

The magnitude of the seismic force also depends on the type of foundation soil under the building. Some soils tend to amplify seismic waves and can even tend to liquefy during an earthquake. Hence, it is important to suitably incorporate the effect of prevailing soil conditions in the procedures of evaluation of seismic forces on the structure.

Introduction of an occupancy importance factor to provide for more conservative design of important facilities is necessary such that the structure importance factor indirectly accounts for less risk, or better expected performance specified for important structures. Important structures are those

- Emergency facilities that are expected to remain functional after a severe earthquake such as hospitals, fire stations, etc.
- Buildings, whose failure may lead to other disasters, affecting people or environment, such as nuclear power plants, dams, petrochemical facilities, etc.
- Life-line facilities e.g. communication lines, pipelines, bridges, power stations, etc.
- Facilities for large number of people such as community centers, schools, etc.

Accordingly these structures are designed for higher lateral strength, and hence they are expected to sustain less damage under the design earthquake.

Finally, it is imperative to rationally incorporate means of reducing the required lateral strength in case of structures that are capable of withstanding extensive inelastic behaviour by virtue of their structural configuration and detailing. In this regard, generally provision is made in the code of practice by introducing the response modification factor. The response reduction factor essentially reduces the design lateral strength of the structure from required strength to resist the linear response to the strength that would be required to limit inelastic behaviour to acceptable levels. Response reduction factor magnitude mainly depends on the ductility characteristics of the structure under consideration. Structural systems deemed capable of withstanding extensive inelastic behavior are assigned relatively high response reduction factor values, permitting minimum design strength that is required for elastic response to the design ground motion. Systems deemed to be incapable of providing reliable inelastic behavior are assigned with low response reduction factor value that results in strength sufficient to resist design motion in a nearly elastic manner.

4 General Earthquake Resistant Design Principles of IS-1893 (2002)

Clause 6.1 of IS-1893 (2002) provides the following design principles,

- The random earthquake ground motions, which cause the structure to vibrate, can be resolved in any three mutually perpendicular directions. The predominant direction of ground vibration is usually horizontal.
- Earthquake-generated vertical inertia forces are to be considered in design unless checked and proven in specimen calculations to be not significant. Vertical acceleration should be considered in structures with large spans and those in which stability is a criterion for design. Reduction in gravity force due to vertical component of ground motions can be particularly detrimental in cases of prestressed horizontal members and of cantilevered members. Hence, special attention should be paid to the effect of vertical component of the ground motion on prestressed or cantilevered beams, girders and slabs.
- The response of a structure to ground vibration is a function of the nature of foundation soil: materials, form, size and mode of construction of structures and the duration and characteristics of ground motion. IS-1893 specifies design forces for structures standing on rocks or soils which do not settle or liquefy or slide due to loss of strength during ground vibrations.
- The design approach adopted in IS 1893 ensures that structures possess at least a minimum strength to withstand minor earthquakes of intensity less than DBE (Design Basis Earthquake) without damage; resist moderate earthquakes equal to DBE without significant structural damage though some non-structural damage may occur; and aims that structures withstand a major earthquake (Maximum Considered Earthquake - MCE) without collapse.
- Actual forces that appear on structures during earthquakes are much greater than the design forces specified in the code. However, ductility, arising from inelastic material behaviour and detailing, and over strength, arising from the additional reserve strength in structures over and above the design strength, are relied upon to account for this difference in actual and design lateral loads.
- The design lateral force specified in this standard shall be considered in each of the two orthogonal horizontal directions of the structure. For structures which have lateral force resisting elements in the two orthogonal directions only, the design lateral force shall be

considered along one direction at a time, and not in both directions simultaneously. Structures, having lateral force resisting elements (for example frames, shear walls) in directions other than the two orthogonal directions, shall be analysed considering the load combinations specified in Clause: 6.3.2 [IS-1893 (2002)]. Where both horizontal and vertical seismic forces are taken into account, load combinations specified in Clause: 6.3.3 [IS-1893 (2002)] shall be considered. (Refer to equation (3) & (4) for load combinations specified in IS-1893)

5 Assumptions Made Earthquake Resistant Design of Structures

The following assumptions are made in IS-1893 (2002) for earthquake resistant design of structures (Clause: 6.2, IS 1893-2002):

- Earthquake causes impulsive ground motions, which are complex and irregular in character, changing in period and amplitude each lasting for a small duration. Therefore, resonance of the type as visualised under steady-state sinusoidal excitations, will not occur as it would need time to build up such amplitudes
- Earthquake is not likely to occur simultaneously with wind or maximum flood or maximum sea waves.
- The value of elastic modulus of materials, wherever required, may be taken as for static analysis unless a more definite value is available for use in such condition

6 Load combinations

Clause: 6.3 of IS-1893 (2002) specifies following load combinations

- In the plastic design of steel structures, the following load combinations shall be accounted for:

$$\begin{aligned} &1)\ 1.7(DL+IL) \\ &2)\ 1.7(DL\pm IL) \qquad (1) \\ &3)\ 1.3(DL+IL\pm EL) \end{aligned}$$

- In the limit state design of reinforced and prestressed concrete structures, the following load combinations shall be accounted for:

$$
\begin{aligned}
&1)\ 1.5(DL + IL) \\
&2)\ 1.2(DL + IL \pm EL) \\
&3)\ 1.5(DL \pm IL) \\
&4)\ 0.9DL \pm 1.5EL
\end{aligned}
\qquad (2)
$$

Where DL, IL and EL denote dead load, imposed load and earthquake load respectively.

- Design Horizontal Earthquake Load:
  - When the lateral load resisting elements are oriented along orthogonal horizontal direction, the structure shall be designed for the effects due to full design earthquake load in one horizontal direction at time.
  - When the lateral load resisting elements are not oriented along the orthogonal horizontal directions, the structure shall be designed for the effects due to full design earthquake load in one horizontal direction plus 30 percent of the design earthquake load in the other direction
- Design Vertical Earthquake Load: When effects due to vertical earthquake loads are to be considered, the design vertical force shall be calculated in accordance with Clause: 6.4.5 of IS-1893 (2002). (i.e., the design acceleration spectrum for vertical motions may be taken as two-thirds of the design horizontal acceleration spectrum)
- Combination for Two or Three Component Motion: When responses from the three earthquake components are to be considered, the responses due to each component may be combined using the assumption that when the maximum response from one component occurs, the responses from the other two components are 30 percent of their maximum. All possible combinations of the three components (EL_x, EL_y and EL_z where x and y are two orthogonal directions and z is vertical direction) including variations in sign (plus or minus) shall be considered. Thus, the response due earthquake force (EL) is the maximum of the following three cases (Clause: 6.3.4.1, IS 1893-2002)

$$
\begin{aligned}
&1)\ \pm EL_x \pm 0.3EL_y \pm 0.3EL_z \\
&2)\ \pm 0.3EL_x \pm EL_y \pm 0.3EL_z \\
&3)\ \pm 0.3EL_x \pm 0.3EL_y \pm EL_z
\end{aligned}
\qquad (3)
$$

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Or as an alternative to the procedure mentioned above, the response *(EL)* due to the combined effect of the three components can be obtained (Clause: 6.3.4.2, IS 1893-2002) on the basis SRSS that is,

$$EL = \sqrt{(EL_x)^2 + (EL_y)^2 + (EL_z)^2} \qquad (4)$$

7 Design Spectrum

- For the purpose of determining seismic forces, the country is classified into four seismic zones as shown in Figure 2.

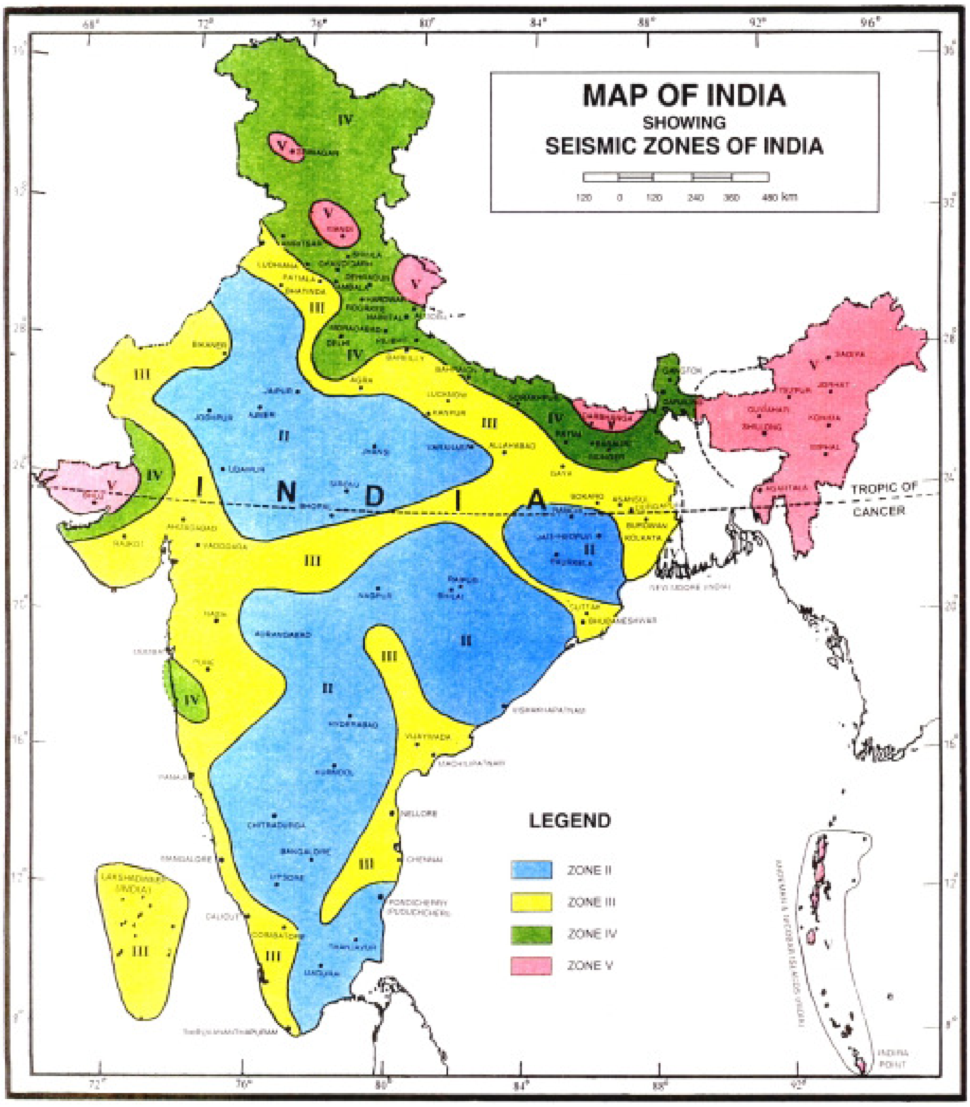


Figure 2: Seismic zones of India [Fig. 1, IS-1893 (2002)]

- The design horizontal seismic coefficient for a structure shall be determined by the following expression (Clause: 6.4.2.1, IS 1893-2002)

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} \qquad (5)$$

Provided that for any structure with T ≤ 0.1 s, the value of A_h will not be taken less than $Z/2$ whatever be the value of I/R. Where,

Z = Zone factor given in Table 1, is for the Maximum Considered Earthquake (MCE) and service life of structure in a zone. The factor 2 in the denominator of Z is used so as to reduce the Maximum Considered Earthquake (MCE) zone factor to the factor for Design Basis Earthquake (DBE).

I = Importance factor, depending upon the functional use of the structures, characterised by hazardous consequences of its failure, post earthquake functional needs, historical value, or economic importance (Table 2).

(S_a / g) = Average response acceleration coefficient for rock or soil sites as given by Figure 3 (or from table adjacent to the Figure 3) based on appropriate natural periods and damping of the structure. These curves represent free field ground motion. Figure 3 shows the proposed 5% spectra for rocky and soils sites and Table 3 gives the multiplying factors for obtaining spectral values for various other damping.

R = Response reduction factor, depending on the perceived seismic damage performance of the structure, characterised by ductile or brittle deformations. However, the ratio *(I/R)* shall not be greater than 1.0. The values of *R* for buildings are given in Table 4.

- Where a number of modes are to be considered for dynamic analysis, the value of A_h as defined in equation (5), for each mode shall be determined using the natural period of vibration of that mode.
- For underground structures and foundations at depths of 30 m or below, the design horizontal acceleration spectrum value shall be taken as half the value obtained from equation (5). For structures and foundations placed between the ground level and 30 m depth, the design horizontal acceleration spectrum value shall be linearly interpolated between A_h and $0.5A_h$ where A_h is as specified in equation (5)

Table 1: Zone factor (*Z*) [Table 2, IS-1893 (2002)]

Seismic Zone	**II**	**III**	**IV**	**V**
Seismic Intensity	Low	Moderate	Severe	Very Severe
Z	0.10	0.16	0.24	0.36

Table 2: Importance factor (*I*) [Table 6, IS-1893 (2002)]

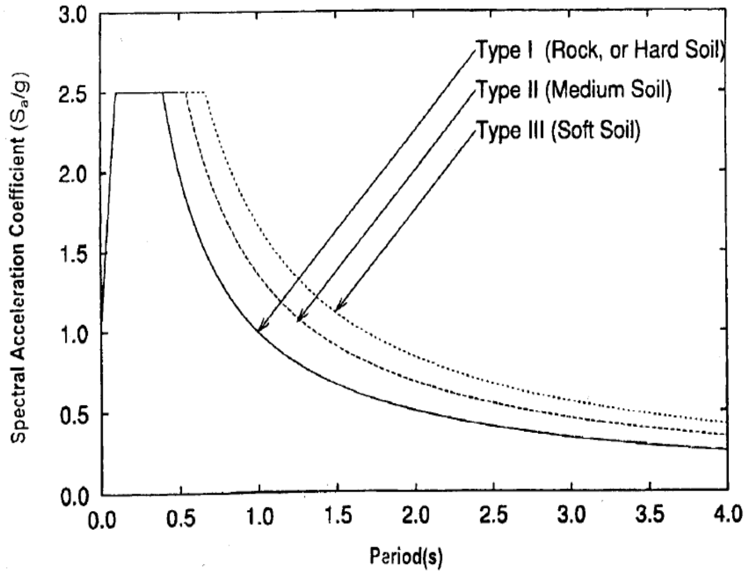
SI. No. (1)	**Structure** (2)	**Importance Factor** (3)
i)	Important service and community buildings, such as hospitals; schools; monumental structures; emergency buildings like telephone exchange, television stations, radio stations, railway stations, tire station buildings; large community halls like cinemas, assembly halls and subway stations, power stations	1.5
ii)	All other buildings	1.0

NOTES

1. The design engineer may choose values of importance factor *I* greater than those mentioned above.

2. Buildings not covered in SI. No. (i) and (ii) above may be designed for higher value of *I*. depending on economy, strategy considerations like multi-storey buildings having several residential units

3. This docs not apply to temporary structures like excavations, scaffolding etc of short duration.



For rocky, or hard soil sites

$$\frac{S_a}{g} = \begin{cases} 1+15T & 0.00 \le T \le 0.10 \\ 2.50 & 0.10 \le T \le 0.40 \\ 1.00/T & 0.40 \le T \le 4.00 \end{cases}$$

For medium soil sites

$$\frac{S_a}{g} = \begin{cases} 1+15T & 0.00 \le T \le 0.10 \\ 2.50 & 0.10 \le T \le 0.55 \\ 1.36/T & 0.55 \le T \le 4.00 \end{cases}$$

For soft soil sites

$$\frac{S_a}{g} = \begin{cases} 1+15T & 0.00 \le T \le 0.10 \\ 2.50 & 0.10 \le T \le 0.67 \\ 1.67/T & 0.67 \le T \le 4.00 \end{cases}$$

Figure 3: Response spectra for rock and soil sites for 5% damping [Fig. 2, IS-1893 (2002)]

Table 3 Multiplying factors for damping other than 5% [Table 3, IS-1893 (2002)]

Damping Percent	0	2	5	7	10	15	20	25	30
Factors	3.20	1.40	1.00	0.90	0.80	0.70	0.60	0.55	0.50

Table 4: Response reduction factor (*R*) for building systems [Table 7, IS-1893 (2002)]

SI. No.	Lateral Load Resisting System	*R*
(1)	(2)	(3)
	Building Frame Systems	
(i)	Ordinary RC moment-resisting frame (OMRF)[2)]	3.0
(ii)	Special RC moment-resisting frame (SMRF)[3)]	5.0
(iii)	Steel frame with	
	a) Concentric braces	4.0
	b) Eccentric braces	5.0
(iv)	Steel moment resisting frame designed as per SP 6 (6)	5.0
	Building with Shear Walls[4)]	
(v)	Load bearing masonry wall buildings[5)]	
	a) Unreinforced	1.5
	b) Reinforced with horizontal RC bands	2.5
	c) Reinforced with horizontal RC bands and vertical bars at comers of rooms and	3.0
(vi)	Ordinary reinforced concrete shear walls[6)]	3.0
(vii)	Ductile shear walls[7)]	4.0
	Buildings with Dual System[8)]	
(viii)	Ordinary shear wall with OMRF	3.0
(ix)	Ordinary shear wall with SMRF	4.0
(x)	Ductile shear wall with OMRF	4.5
(xi)	Ductile shear wall with SMRF	5.0

1) The values of response reduction factors are to be used for buildings with lateral load resisting elements, and not just for the lateral load resisting elements built in isolation.
2) OMRF are those designed and detailed as per IS 456 or IS 800 but not mooting ductile detailing requirement as per IS 13920 or SP 6 (6) respectively.
3) SMRF defined in 4.15.2.
4) Buildings with shear walls also include buildings having shear walls and frames, but where:

a) frames are not designed to carry lateral loads, or
b) frames are designed to carry lateral loads but do not fulfil the requirements of 'dual systems'.

5) Reinforcement should be as per IS 4326.
6) Prohibited in zones IV and V.
7) Ductile shear walls are those designed and detailed as per IS 13920.
8) Buildings with dual systems consist of shear walls (or braced frames) and moment resisting frames such that:

a) the two systems arc designed to resist the total design force in proportion to their lateral stiffness considering the interaction of the dual system at all floor levels; and
b) the moment resisting frames are designed to independently resist at least 25 percent of the design seismic base shear.

8 Design imposed loads for earthquake force calculation [Clause 7.3, IS 1893 (2002)]

- For various loading classes as specified in IS 875 (Part 2), the earthquake force shall be calculated for the full dead load plus the percentage of imposed load as given in Table 5.

Table 5: Percentage of imposed load to be considered in seismic weight calculation [Table 8, IS-1893 (2002)]

Imposed Uniformly Distributed Floor Loads (kN/m^2)	Percentage of Imposed Load
Up to and including 3.0	25
Above 3.0	50

- For calculating the design seismic forces of the structure, the imposed load on roof need not be considered.
- The percentage of imposed loads given above shall also be used for 'Whole frame loaded' condition in the load combinations specified in equation (2) and equation (3) where the gravity loads are combined with the earthquake loads. No further reduction in the imposed load will be used as envisaged in IS 875 (Part 2) for number of storeys above the one under consideration or for large spans of beams or floors.
- The proportions of imposed load indicated above for calculating the lateral design forces for earthquakes are applicable to average conditions. Where the probable loads at the time of earthquake are more accurately assessed, the designer may alter the proportions indicated or even replace the entire imposed load proportions by the actual assessed load. In such cases, where the imposed load is not assessed as mentioned above only that part of imposed load, which possesses mass, shall be considered. Lateral design force for earthquakes shall not be calculated on contribution of impact effects from imposed loads.
- Other loads apart from those given above (for example snow and permanent equipment) shall be considered as appropriate.

9 Design lateral forces

Design Seismic Base Shear: The total design lateral force or design seismic base shear (V_B) along any principal direction shall be determined by the following expression: [Clause 7.5.3, IS-1893 (2002)]

$$V_B = A_h W \qquad (6)$$

Where

A_h = Design horizontal acceleration spectrum value as per equation (5), using the fundamental natural period T_a as per equation (7) or (8) in the considered direction of vibration; and

W = Seismic weight of the building is computed as given below [Clauses 7.4.2 & 7.4.3, IS-1893 (2002)]

- *Seismic Weight of floors:* The seismic weight of each floor is its full dead load plus appropriate amount of imposed load. While computing the seismic weight of each floor, the weight of columns and walls in any storey shall be equally distributed to the floors above and below the storey.
- *Seismic Weight of Building:*
  - The seismic weight of the whole building is the sum of the seismic weights of all the floors.
  - Any weight supported in between storeys shall be distributed to the floors above and below in inverse proportion to its distance from the floors.

10 Fundamental period

- The approximate fundamental natural period of vibration (T_a), in seconds, of a moment-resisting frame building without brick infill panels may be estimated by the empirical expression: [Clause 7.6.1, IS-1893 (2002)]

$$\begin{aligned} T_a &= 0.075h^{0.75} \text{....(for RC frame building)} \\ T_a &= 0.085h^{0.75} \text{....(for steel frame building)} \end{aligned} \qquad (7)$$

- The approximate fundamental natural period of vibration (T_a), in seconds, of all other buildings, including moment-resisting frame buildings with brick infill panels, may be estimated by the empirical expression: [Clause 7.6.2, IS-1893 (2002)]

$$T_a = \frac{0.09h}{\sqrt{d}} \tag{8}$$

where,

h = Height of building, in m. This excludes the basement storeys, where basement walls are connected with the ground floor deck or fitted between the building columns. But, it includes the basement storeys, when they are not so connected.

d = Base dimension of the building at the plinth level, in m, along the considered direction of the lateral force.

11 Earthquake Lateral Force Analysis

The design lateral force shall first be computed for the building as a whole. This design lateral force shall then be distributed to the various floor levels. The overall design seismic force thus obtained at each floor level shall then be distributed to individual lateral load resisting elements depending on the floor diaphragm action. There are two commonly used procedures for specifying seismic design lateral forces:

1. Equivalent static force analysis
2. Dynamic analysis

12 Equivalent static force analysis

The equivalent lateral force for an earthquake is a unique concept used in earthquake engineering. The concept is attractive because it converts a dynamic analysis into partly dynamic and partly static analyses for finding the maximum displacement (or stresses) induced in the structure due to earthquake excitation. For seismic resistant design of structures, only these maximum stresses are of interest, not the time history of stresses. The equivalent lateral force for an earthquake is defined as a set of lateral static forces which will produce the same peak response of the structure as that obtained by the dynamic analysis of the structure under the same earthquake. This equivalence is restricted only to a single mode of vibration of the structure. Inherently, equivalent static lateral force analysis is based on the following assumptions,

- Assume that structure is rigid.
- Assume perfect fixity between structure and foundation.
- During ground motion every point on the structure experience same accelerations
- Dominant effect of earthquake is equivalent to horizontal force of varying magnitude over the height.
- Approximately determines the total horizontal force (Base shear) on the structure

However, during an earthquake structure does not remain rigid, it deflects, and thus base shear is disturbed along the height.

The limitations of equivalent static lateral force analysis may be summarised as follows,

- In the equivalent static force procedure, empirical relationships are used to specify dynamic inertial forces as static forces.
- These empirical formulas do not explicitly account for the dynamic characteristics of the particular structure being designed or analyzed.
- These formulas were developed to approximately represent the dynamic behavior of what are called *regular* structures (Structures which have a reasonably uniform distribution of mass and stiffness). For such structures, the equivalent static force procedure is most often adequate.
- Structures that are classified as *irregular* violate the assumptions on which the empirical formulas, used in the equivalent static force procedure, are developed. Common types of irregularities in a structure include large floor-to-floor variation in mass or center of mass and soft stories etc. Therefore in such cases, use of equivalent static force procedure may lead to erroneous results. In these cases, a dynamic analysis should be used to specify and distribute the seismic design forces.

13 Step by step procedure for Equivalent static force analysis

Step-1: Depending on the location of the building site, identify the seismic zone and assign Zone factor (*Z*)

- Use Table 2 along with Seismic zones map or Annex of IS-1893 (2002)

Step-2: Compute the seismic weight of the building (*W*)

- As per Clause 7.4.2, IS-1893 (2002) – Seismic weight of floors
- As per Clause 7.4.3, IS-1893 (2002) – Seismic weight of the building

Step-3: Compute the natural period of the building (T_a)

- As per Clause 7.6.1 or Clause 7.6.2, IS-1893 (2002), as the case may be.

Step-4: Obtain the data pertaining to type of soil conditions of foundation of the building

- Assign type, I for hard soil, II for medium soil & III for soft soil

Step-5: Using T_a and soil type (I / II / III), compute the average spectral acceleration $\left(\frac{S_a}{g}\right)$

- Use Figure 2 or corresponding table of IS-1893 (2002), to compute S_a/g

Step-6: Assign the value of importance factor (I) depending on occupancy and/or functionality of structure

- As per Clause 7.2 and Table 6 of IS-1893 (2002),

Step-7: Assign the values of response reduction factor (R) depending on type of structure

- As per Clause 7.2 and Table 7 of IS-1893 (2002)

Step-8: Knowing Z, S_a/g, R and I compute design horizontal acceleration coefficient (A_h) using the relationship, $A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R}$ [Clause 6.4.2, IS-1893 (2002)]

Step-9: Using A_h and W compute design seismic base shear (V_B), from $V_B = A_h W$ [Clause 7.5.3, IS-1893 (2002)]

Step-10: Compute design lateral force (Q_i) of i^{th} floor by distributing the design seismic base shear (V_B) as per the expression, $Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}$ [Clause 7.7.1, IS-1893 (2002)]

14 Dynamic Analysis

- Dynamic analysis is classified into two types, namely, *Response spectrum method* and *Time history method*
- Dynamic analysis shall be performed to obtain the design seismic force, and its distribution to different levels along the height of the building and to the various lateral load resisting elements, for the following buildings:

a) *Regular buildings* — Those greater than 40 m in height in Zones IV and V, and those greater than 90 m in height in Zones II and III.
b) *Irregular buildings* — All framed buildings higher than 12 m in Zones IV and V, and those greater than 40 m in height in Zones II and III.

- *Time History Method:* Time history method of analysis, when used, shall be based on an appropriate ground motion and shall be performed using accepted principles of dynamics.
- *Response Spectrum Method:* Response spectrum method of analysis shall be performed using the design spectrum specified in Clause 6.4.2 or by a site specific design, spectrum mentioned in Clause 6.4.6 of IS 1893 (2002)
- When dynamic analysis is carried out either by the Time History Method or by the Response Spectrum Method, the design base shear computed from dynamic analysis (V_B) shall be compared with a base shear calculated using a fundamental period T_a ($\bar{V}_B$), where T_a is as per Clause 7.6. If base shear obtained from dynamic analysis (V_B) is less than base shear computed from equivalent static load method ($\bar{V}_B$ *i.e., using* T_a *as per Clause 7.6*), then as per Clause 7.8.2, all the response quantities (for example member forces, displacements, storey forces, storey shears and base reactions) shall be multiplied by ratio $\frac{\bar{V}_B}{V_B}$.
- *Free Vibration Analysis:* Undamped free vibration analysis of the entire building shall be performed as per established methods of mechanics using the appropriate masses and elastic stiffness of the structural system, to obtain natural periods (T) and mode shapes (φ) of those of its modes of vibration that need to be considered.
- *Modes to be considered:* The number of modes to be used in the analysis should be such that the sum total of modal masses of all modes considered is at least 90%. If modes with natural frequency beyond 33 *Hz* are to be considered, modal combination shall be carried out only for modes up to 33 *Hz*. The effect of modes with natural frequency beyond 33 *Hz* be included by considering missing mass correction following well established procedures.

- *Modal combination:* The peak response quantities (for example, member forces, displacements, storey forces, storey shears and base reactions) shall be combined as per Complete Quadratic Combination (CQC) method or alternatively, when building does not have closely spaced modes then Square Root of Square Sum (SRSS) method may be employed.

CQC method: $$\lambda = \sqrt{\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i \rho_{ij} \lambda_j} \quad (9)$$

Where,

r = Number of modes being considered,

$$\rho_{ij} = \frac{8\varsigma^2 (1+\beta_{ij})\beta_{ij}^{1.5}}{(1-\beta_{ij}^2)^2 + 4\varsigma^2 \beta_{ij}(1+\beta_{ij})^2}$$ = Cross-modal coefficient,

λ_i = Response quantity in mode i(including sign),

λ_j = Response quantity in mode j(including sign),

β_{ij} = frequency ratio between i^{th} and the j^{th} mode is, $\beta_{ij} = \frac{\omega_j}{\omega_i} = \frac{T_j}{T_i}$

SRSS method: $$\lambda = \sqrt{\sum_{k=1}^{r}\lambda_k^2} \quad (10)$$

λ_k = Absolute value of response quantity in mode k

15 Step by step procedure for Response spectrum method

Step-1: Depending on the location of the building site, identify the seismic zone and assign Zone factor (Z)

- Use Table 2 along with Seismic zones map or Annex of IS-1893 (2002)

Step-2: Compute the seismic weight of the building (W)

- As per Clause 7.4.2, IS-1893 (2002) – Seismic weight of floors (W_i)

Step-3: Establish mass *[M]* and stiffness *[K]* matrices of the building using system of masses lumped at the floor levels with each mass having one degree of freedom, that of lateral displacement in the direction under consideration. Accordingly, to develop stiffness matrix effective stiffness of each floor is computed using the lateral stiffness coefficients of columns and infill walls. Usually floor slab is assumed to be infinitely stiff.

Step-4: Using *[M]* and *[K]* of previous step and employing the principles of dynamics compute the modal frequencies, $\{\omega\}$ and corresponding mode shapes, $[\varphi]$.

Step-5: Compute modal mass M_k of mode k using the following relationship with n being number of modes considered

$$M_k = \frac{\left[\sum_{i=1}^{n} W_i \phi_{ik}\right]^2}{g \sum_{i=1}^{n} W_i \phi_{ik}^2} \qquad \text{[Clause 7.8.4.5a of IS 1893 (2002)]} \qquad (11)$$

Step-6: Compute modal participation factors P_k of mode k using the following relationship with n being number of modes considered

$$P_k = \frac{\sum_{i=1}^{n} W_i \phi_{ik}}{\sum_{i=1}^{n} W_i \phi_{ik}^2} \qquad \text{[Clause 7.8.4.5b of IS 1893 (2002)]} \qquad (12)$$

Step-7: Compute design lateral force (Q_{ik}) at each floor in each mode (i.e., for i^{th} floor in mode k) using the following relationship,

$$Q_{ik} = A_{h(k)}\ \phi_{ik}\ P_k\ W_i \qquad \text{[Clause 7.8.4.5c of IS 1893 (2002)]} \qquad (13)$$

$A_{h(k)}$ = Design horizontal acceleration spectrum value as per Clause 6.4.2 of IS 1893 using the natural period $\left(T_k = \dfrac{2\pi}{\omega_k}\right)$ of vibration of mode k.

Step-8: Compute storey shear forces in each mode (V_{ik}) acting in storey i in mode k as given by,

$$V_{ik} = \sum_{i+1}^{n} Q_{ik} \qquad \text{[Clause 7.8.4.5d of IS 1893 (2002)]} \qquad (14)$$

Step-9: Compute storey shear forces due to all modes considered, V_i in storey i, by combining shear forces due to each mode in accordance with Clause 7.8.4.4 of IS 1893 (2002). i.e., either CQC or SRSS modal combination methods are used.

Step-10: Finally compute design lateral forces at each storey as,

$$F_{roof} = V_{roof} \text{ and} \qquad \text{[Clause 7.8.4.5f of IS 1893 (2002)]} \qquad (15)$$
$$F_i = V_i - V_{i+1}$$

EXAMPLE: 1

Plan and elevation of a four-storey reinforced concrete office building is shown in Fig. 1.1. The details of the building are as follows.

Number of Storey = 4

Zone = III

Live Load = 3 kN/m^2

Columns = 450 x 450 mm

Beams = 250 x 400 mm

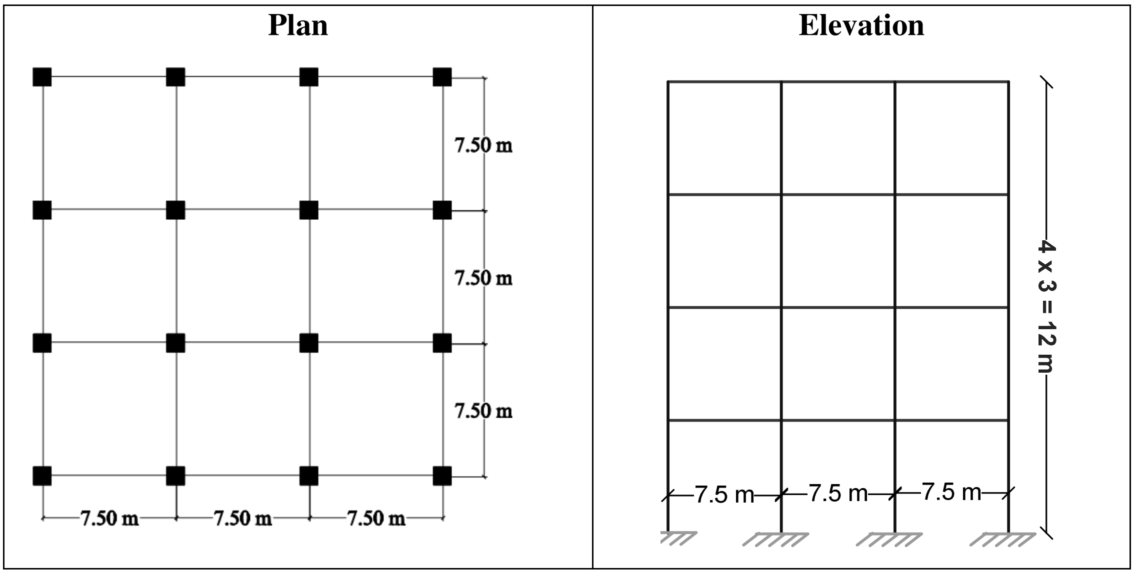
Thickness of Slab = 150 mm

Thickness of Wall = 120 mm

Importance factor = 1.0

Structure type = OMRF Building

Determine design seismic lateral load and storey shear force distribution.



Solution: Analysis considering stiffness of infill masonry

1. Computation of Seismic weights

(Assuming unit weight of concrete as 25 kN/m^3 & 22.5 kN/m^3 for masonry)

1) Slab:

DL due to self weight of slab = (22.5 x 22.5 x 0.15) x 25 = 1898.40 kN

2) Beams:

Self weight of beam per unit length = 0.25 x 0.4 x 25 = 2.5 kN/m

Total length = 4x22.5x2 = 180 m

DL due to self weight of beams = (2.5 x 22.5) x 4 x 2 = 450 kN

3) Columns:

Self weight of column per unit length = 0.45 x 0.45 x 25 = 5.0625 kN/m

DL due to self weight of columns (16 No.s) = 16 x 5.0625 x 3.0 = 243 kN

4) Walls:

Self weight of wall per unit length = 0.12 x 3 x 20 = 7.2 kN/m

Total length = 4x22.5x2 = 180 m

DL due to self weight of Walls = 7.2 x 22.5 x 4 = 648 kN

5) Live Load [Imposed load] (25 %) = (0.25 x 3) x 22.5 x 22.5 = 380 kN

Load on all floors:

W1=W2=W3= 1898 + 380 + 450 + 243 + 648 = **3619 kN**

Load on roof slab (Live load on slab is zero)

W4 = 1898 + 0+450 + (243/2) + (648/2) = **2793.5 kN**

Total Seismic weight, W = (3619 x 3) + 2793.5 = **13650.5 kN**

Fundamental period:

Natural period, $T_a = 0.09\frac{h}{\sqrt{d}} = 0.09\frac{12}{\sqrt{22.5}} = 0.2277$

(Moment resisting frame with in-fill walls)

Spectral acceleration:

Type of soil: Medium Soil

For T_a=0.2277 s

Sa/g = 2.5

Zone factor: For Zone III, Z = 0.16

Importance Factor: I = 1.0

Response Reduction Factor: R = 3.0 (OMRF)

Horizontal acceleration coefficient (A_h):

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.16}{2}(2.5)\left(\frac{1}{3}\right)$$

$$A_h = 0.0667$$

Base shear (V_B):

$$V_B = A_h W = 0.0667 \times 13650.50$$

$$V_B = 910.0333\ kN$$

Storey lateral forces and shear forces are calculated and tabulated in the following table.

Floor level (i)	$W_i\,(kN)$	$h_i\,(m)$	$W_i h_i^2$ (kN-m2)	Storey forces $Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}$	Storey shear forces [V_i] (Cumulative sum) (kN)
4	2793.5	12.0	402264	426.53	426.53
3	3619	9.0	293139	310.83	737.35
2	3619	6.0	130284	138.14	875.50
1	3619	3.0	32571	34.54	910.03

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Storey shear forces are calculated as follows (last column of the table),

$$V_4 = Q_4 = 426.53\ kN$$
$$V_3 = V_4 + Q_3 = 426.53{+}310.82{=}737.35\ kN$$
$$V_2 = V_3 + Q_2 = 737.35{+}138.14{=}875.50\ kN$$
$$V_1 = V_2 + Q_1 = 875.50{+}34.54{=}\ 910.03\ kN = V_B$$

Later force and shear force distribution is shown in the Figure-EX1.

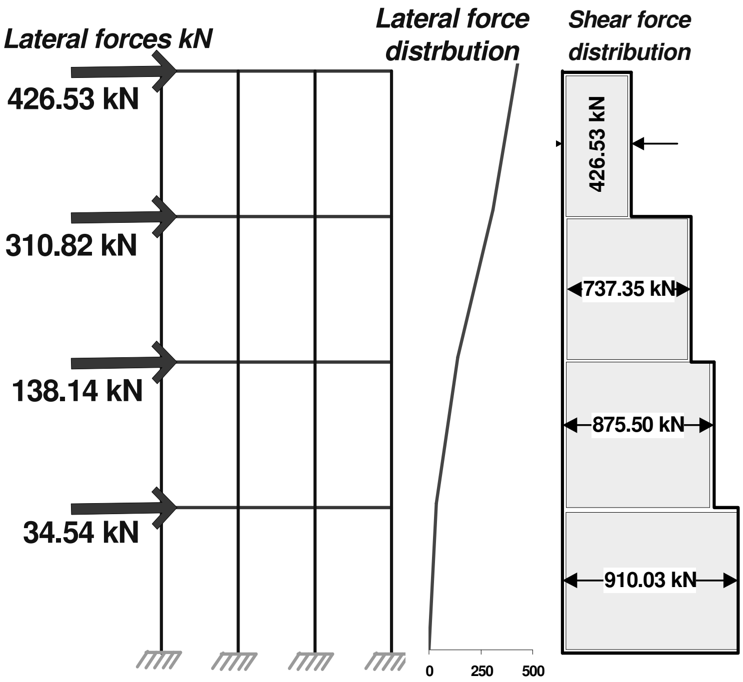

Figure – EX1: Lateral and Shear Force distribution along the height of the structure

Solution: Analysis without considering stiffness of infill masonry

Fundamental period:

Natural period, $T_a = 0.075h^{0.75} = 0.075 \times 12^{0.75} = 0.4836$
(Moment resisting frame without in-fill walls)

Spectral acceleration:

Type of soil: Medium Soil

For $T_a = 0.4836$ s

Sa/g = 2.5 (because, $T_a = 0.4836$ s, i.e., $0.10 \le T_a \le 0.55$)

Zone factor: For Zone III, Z = 0.16

Importance Factor: I = 1.0

Response Reduction Factor: R = 3.0 (OMRF)

Horizontal acceleration coefficient (A_h) :

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.16}{2}(2.5)\left(\frac{1}{3}\right)$$

$$A_h = 0.0667$$

Base shear (V_B) :

$$V_B = A_h W = 0.0667 \times 13650.50$$

$$V_B = 910.0333 \ kN$$

Since, base shear V_B is same as in case of considering stiffness of infill walls, the storey lateral forces and shear forces are same as in the previous case. Therefore, Lateral and Shear Force distribution along the height of the structure shown in Figure-EX1 is valid. That is, for the structure under consideration, the lateral force and shear force distribution is unaltered irrespective of stiffness of infill walls is included or not in the analysis.

EXAMPLE: 2

Analyse the building frame considered in Example-1 using response spectrum method (Dynamic analysis) with all other data being same.

Solution:

Note: In plan structure is symmetrical about both X and Y directions)

1) Seismic weights:

   W1=W2=W3= 1898 + 380 + 450 + 243 + 648 = **3619 kN**

   W4 = 1898 + 0+450 + (243/2) + (648/2) = **2793.5 kN**

   Therefore, seismic masses are,

   M1 = M2 = M3 = 368.91 x 10^3 kg.

   M4 = 284.76 x 10^3 kg

2) Floor stiffness (Without considering stiffness of infill wall):

   MI of columns, $I_C = (0.45)^4/12 = 3.1417875 \times 10^{-3}$ m^4

Young's Modulus, $E_C = 5000(f_{ck})^{0.5} = 25000$ MPa = 25×10^9 N/m^2

(Assuming M25 concrete for columns)

K1 = K2 = K3 = K4 = $16 \times (12 \times 25 \times 10^9 \times 3.1417875 \times 10^{-3})/(3^3) = 0.6075 \times 10^9$ N/m

3) Natural frequencies and Mode shapes:

Mass matrix,

$$M = \begin{bmatrix} M1 & 0 & 0 & 0 \\ 0 & M2 & 0 & 0 \\ 0 & 0 & M3 & 0 \\ 0 & 0 & 0 & M4 \end{bmatrix} = \begin{bmatrix} 368.91 & 0 & 0 & 0 \\ 0 & 368.91 & 0 & 0 \\ 0 & 0 & 368.91 & 0 \\ 0 & 0 & 0 & 284.76 \end{bmatrix} \times 10^3 kg$$

Stiffness Matrix,

$$K = \begin{bmatrix} K1+K2 & -K2 & 0 & 0 \\ -K2 & K2+K3 & -K3 & 0 \\ 0 & -K3 & K3+K4 & -K4 \\ 0 & 0 & -K4 & -K4 \end{bmatrix} = \begin{bmatrix} 1.215 & -0.6075 & 0 & 0 \\ -0.6075 & 1.215 & -0.6075 & 0 \\ 0 & -0.6075 & 1.215 & -0.6075 \\ 0 & 0 & -0.6075 & 0.6075 \end{bmatrix} \times 10^9 \; N/m$$

Solving the Eigen equation, $\left| K - M\omega^2 \right| = 0$, we get Eigen value and corresponding Eigen vectors as,

$$\text{Eigen values, } \omega^2 = \begin{Bmatrix} 219.9 \\ 1793.2 \\ 4079.8 \\ 5920.9 \end{Bmatrix} \quad \therefore \text{ the natural frequencies are, } \omega = \begin{Bmatrix} 14.83 \\ 42.35 \\ 63.87 \\ 76.95 \end{Bmatrix} \; rad/s$$

The mode shapes are,

$$\phi_1 = \begin{Bmatrix} 1.00 \\ 1.87 \\ 2.48 \\ 2.77 \end{Bmatrix}, \; \phi_2 = \begin{Bmatrix} 1.00 \\ 0.91 \\ -0.17 \\ -1.07 \end{Bmatrix}, \; \phi_3 = \begin{Bmatrix} 1.00 \\ -0.48 \\ -0.77 \\ 0.85 \end{Bmatrix}, \; \& \; \phi_4 = \begin{Bmatrix} 1.00 \\ -1.60 \\ 1.55 \\ -0.87 \end{Bmatrix}$$

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The natural periods are, $T = 2\pi / \{\omega\} = \begin{Bmatrix} 0.424 \\ 0.148 \\ 0.098 \\ 0.082 \end{Bmatrix}$ seconds

Calculation of modal participation factor

Storey Level	Seismic weight (W_i), kN	MODE-1		
		ϕ_{i1}	$W_i\ \phi_{i1}$	$W_i\ \phi_{i1}^{\ 2}$
4	2793.5	2.77	7737.995	21434.25
3	3619	2.48	8975.12	22258.3
2	3619	1.87	6767.53	12655.28
1	3619	1.00	3619.00	3619.00
Σ	**13650.5**		**27099.65**	**59966.82**
Modal mass $M_1 = \dfrac{\left[\sum W_i\phi_{i1}\right]^2}{g\sum W_i\phi_{i1}^2}$		$= \dfrac{27099.65^2}{59966.82g} =$ 12246.62 kN/g		
% of Total weight		89.72 %		
Modal participation factor, $P_1 = \dfrac{\sum W_i\phi_{i1}}{\sum W_i\phi_{i1}^2}$		$= \dfrac{27099.65}{59966.82} =$ **0.452**		

Storey Level	Seismic weight (W_i), kN	MODE-2		
		ϕ_{i2}	$W_i\ \phi_{i2}$	$W_i\ \phi_{i2}^{\ 2}$
4	2793.5	-1.07	-2989.05	3198.278
3	3619	-0.17	-615.23	104.5891
2	3619	0.91	3293.29	2996.894
1	3619	1.00	3619.00	3619.00
Σ	**13650.5**		**3308.015**	**9918.761**
Modal mass, $M_2 = \dfrac{\left[\sum W_i\phi_{i2}\right]^2}{g\sum W_i\phi_{i2}^2}$		$= \dfrac{3308.015^2}{9918.761g} =$ 1103.26 kN/g		
% of Total weight		8.08 %		
Modal participation factor, $P_2 = \dfrac{\sum W_i\phi_{i2}}{\sum W_i\phi_{i2}^2}$		$= \dfrac{3308.015}{9918.761} =$ **0.334**		

Storey Level	Seismic weight (W_i), kN	MODE-3		
		ϕ_{i3}	$W_i\ \phi_{i3}$	$W_i\ \phi_{i3}^{\ 2}$
4	2793.5	0.85	2374.475	2018.304
3	3619	-0.77	-2786.63	2145.705
2	3619	-0.48	-1737.12	833.8176
1	3619	1.00	3619.00	3619.00
Σ	**13650.5**		**1469.725**	**8616.826**
Modal mass, $M_3 = \dfrac{\left[\sum W_i\phi_{i3}\right]^2}{g\sum W_i\phi_{i3}^2}$		$= \dfrac{1469.725^2}{8616.826g} =$ 250.683 kN/g		
% of Total weight		1.84 %		
Modal participation factor, $P_3 = \dfrac{\sum W_i\phi_{i3}}{\sum W_i\phi_{i3}^2}$		$= \dfrac{1469.725}{8616.826} =$ **0.171**		

Storey Level	Seismic weight (W_i), kN	MODE-4		
		ϕ_{i4}	$W_i\ \phi_{i4}$	$W_i\ \phi_{i4}^{\ 2}$
4	2793.5	-0.87	-2430.35	2114.4
3	3619	1.55	5609.45	8694.648
2	3619	-1.60	-5790.4	9264.64
1	3619	1.00	3619.00	3619.00
Σ	**13650.5**		**1007.705**	**23692.69**
Modal mass, $M_4 = \dfrac{\left[\sum W_i\phi_{i4}\right]^2}{g\sum W_i\phi_{i4}^2}$		$= \dfrac{1007.705^2}{23692.69g} =$ 42.86 kN/g		
% of Total weight		0.314 %		
Modal participation factor, $P_4 = \dfrac{\sum W_i\phi_{i4}}{\sum W_i\phi_{i4}^2}$		$= \dfrac{1007.705}{23692.69} =$ **0.043**		

The lateral load Q_{ik} acting at i^{th} floor in the k^{th} mode is,

$Q_{ik} = A_{h(k)}\ \phi_{ik}\ P_k\ W_i$ ……… (Clause 7.8.4.5c of IS: 1893 Part 1)

The value of $A_{h(k)}$ for different modes is obtained from clause 6.4.2.

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MODE-1:

$$T_1 = 0.424 \;\; s$$
$$\frac{S_a}{g} = 2.5....(0.10 \leq T_1 \leq 0.55 - \text{Medium soil})$$
$$A_{h(1)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.16}{2}(2.5)\frac{1}{3} = 0.0667$$
$$Q_{i1} = A_{h(1)} \;\; \phi_{i1} \;\; P_1 \;\; W_i = 0.0667 \times 0.452 \times (\phi_{i1} \;\; W_i) = 0.03015(\phi_{i1} \;\; W_i)$$

MODE-2:

$$T_2 = 0.148 \;\; s$$
$$\frac{S_a}{g} = 2.5....(0.10 \leq T_2 \leq 0.55 - \text{Medium soil})$$
$$A_{h(2)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.16}{2}(2.5)\frac{1}{3} = 0.0667$$
$$Q_{i2} = A_{h(2)} \;\; \phi_{i2} \;\; P_2 \;\; W_i = 0.0667 \times 0.334 \times (\phi_{i2} \;\; W_i) = 0.0223(\phi_{i2} \;\; W_i)$$

MODE-3:

$$T_3 = 0.098 \;\; s$$
$$\frac{S_a}{g} = 1 + 15T_3 = 2.47....(0.00 \leq T_3 \leq 0.10 - \text{Medium soil})$$
$$A_{h(3)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.16}{2}(2.47)\frac{1}{3} = 0.0659, \text{But, } T_3 \leq 0.10,$$
$$\therefore A_{h(3)} = \frac{Z}{2} = 0.08 > 0.0659$$
$$Q_{i3} = A_{h(3)} \;\; \phi_{i3} \;\; P_4 \;\; W_i = 0.08 \times 0.171 \times (\phi_{i3} \; W_i) = 0.01368(\phi_{i3} \; W_i)$$

MODE-4:

$$T_4 = 0.082 \;\; s$$
$$\frac{S_a}{g} = 1 + 15T_4 = 2.23....(0.00 \leq T_4 \leq 0.10 - \text{Medium soil})$$
$$A_{h(4)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.16}{2}(2.23)\frac{1}{3} = 0.0595, \text{But, } T_3 \leq 0.10,$$
$$\therefore A_{h(4)} = \frac{Z}{2} = 0.08 > 0.0595$$
$$Q_{i4} = A_{h(4)} \;\; \phi_{i4} \;\; P_4 \;\; W_i = 0.08 \times 0.043 \times (\phi_{i4} \; W_i) = 0.00344(\phi_{i4} \; W_i)$$

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Lateral load calculation by modal analysis – SRSS method

Storey level	Weight Wi (kN)	**Mode – 1** [$Q_{i1} = 0.03015(\phi_{i1}\ W_i)$]		
		ϕ_{i1}	Q_{i1}	V_{i1}
4	2793.5	2.77	233.30	**233.30**
3	3619	2.48	270.60	**503.90**
2	3619	1.87	204.04	**707.94**
1	3619	1.00	109.11	**817.05**

Storey level	Weight Wi (kN)	**Mode – 2** [$Q_{i2} = 0.0223(\phi_{i2}\ W_i)$]		
		ϕ_{i2}	Q_{i2}	V_{i2}
4	2793.5	-1.07	-66.66	**-66.66**
3	3619	-0.17	-13.72	**-80.38**
2	3619	0.91	73.44	**-6.93**
1	3619	1.00	80.70	**73.77**

Storey level	Weight Wi (kN)	**Mode – 3** [$Q_{i3} = 0.01368(\phi_{i3}\ W_i)$]		
		ϕ_{i3}	Q_{i3}	V_{i3}
4	2793.5	0.85	32.48	**32.48**
3	3619	-0.77	-38.12	**-5.64**
2	3619	-0.48	-23.76	**-29.40**
1	3619	1.00	49.51	**20.11**

Storey level	Weight Wi (kN)	**Mode – 4** [$Q_{i4} = 0.0034(\phi_{i4}\ W_i)$]		
		ϕ_{i4}	Q_{i4}	V_{i4}
4	2793.5	-0.87	-8.26	**-8.26**
3	3619	1.55	19.07	**10.81**
2	3619	-1.6	-19.69	**-8.88**
1	3619	1.00	12.30	**3.43**

SRSS method (Clause 7.8.4.4 – IS1893-2002):

The contribution of different modes are combined by Square Root of the Sum of the Squares (SRSS) using the following relationship, $V_i = \sqrt{V_{i1}^2 + V_{i2}^2 + V_{i3}^2 + V_{i4}^2}$

Then, storey lateral forces are calculated by, $F_i = V_i - V_{i+1}$.

The results obtained are tabulated in the following table.

Storey level	V_{i1}	V_{i2}	V_{i3}	V_{i4}	**Combined shear force (SRSS) V_i (kN)**	**Combined lateral force (SRSS) F_i (kN)**
4	**233.30**	**-66.66**	**32.48**	**-8.26**	**244.94**	**244.94**
3	**503.90**	**-80.38**	**-5.64**	**10.81**	**510.42**	**265.48**
2	**707.94**	**-6.93**	**-29.40**	**-8.88**	**708.64**	**198.22**
1	**817.05**	**73.77**	**20.11**	**3.43**	**820.63**	**111.99**

CQC method (Clause 7.8.4.4 – IS1893-2002):

(Important note: *Since modal frequencies are well separated in this example, the SRSS modal combination method is sufficient to combine contribution of each mode. For the purpose of demonstration CQC method of modal combination and to compare SRSS and CQC methods following calculations are carried out. However, CQC method is preferred when modal frequencies are closely spaced***)**

The contributions of different modes are combined by Complete Quadratic Combination (CQC) method as demonstrated in the following calculations. Shear force quantities in each of the four modes can be expressed as,

$$\lambda_4 = \{V_{41} \quad V_{42} \quad V_{43} \quad V_{44}\} = \{233.30 \quad -66.66 \quad 32.48 \quad -8.26\}$$
$$\lambda_3 = \{V_{31} \quad V_{32} \quad V_{33} \quad V_{34}\} = \{503.90 \quad -80.38 \quad -5.64 \quad 10.81\}$$
$$\lambda_2 = \{V_{21} \quad V_{22} \quad V_{23} \quad V_{24}\} = \{707.94 \quad -6.93 \quad -29.40 \quad -8.88\}$$
$$\lambda_1 = \{V_{11} \quad V_{12} \quad V_{13} \quad V_{14}\} = \{817.05 \quad 73.77 \quad 20.11 \quad 3.43\}$$

Where λ_i is the shear force in the i^{th} mode.

β_{ij} is the frequency ratio between i^{th} and the j^{th} mode is, $\beta_{ij} = \frac{\omega_j}{\omega_i} = \frac{T_j}{T_i}$, hence considering all the four modes, β_{ij} may be expressed in matrix form as,

$$\beta_{ij} = \begin{bmatrix} T_1/T_1 & T_2/T_1 & T_3/T_1 & T_4/T_1 \\ T_1/T_2 & T_2/T_2 & T_3/T_2 & T_4/T_2 \\ T_1/T_3 & T_2/T_3 & T_3/T_3 & T_4/T_3 \\ T_1/T_4 & T_2/T_4 & T_3/T_4 & T_4/T_4 \end{bmatrix} = \begin{bmatrix} 1.00 & 0.35 & 0.23 & 0.19 \\ 2.86 & 1.00 & 0.66 & 0.55 \\ 4.33 & 1.51 & 1.00 & 0.84 \\ 5.17 & 1.80 & 1.20 & 1.00 \end{bmatrix}$$

Where natural periods of different modes are, $T = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 0.424 \\ 0.148 \\ 0.098 \\ 0.082 \end{Bmatrix}$ sec

Now calculate cross modal coefficient ρ_{ij},

$$\rho_{ij} = \frac{8\varsigma^2(1+\beta_{ij})\beta_{ij}^{1.5}}{(1-\beta_{ij}^2)^2 + 4\varsigma^2\beta_{ij}(1+\beta_{ij})^2}$$

Taking damping ratio, $\varsigma = 0.05$ and β_{ij} values computed above, cross modal coefficient ρ_{ij} may be computed and expressed in matrix form as,

$$\rho_{ij} = \begin{bmatrix} 1.0000 & 0.01294 & 0.00460 & 0.00311 \\ 0.09597 & 1.0000 & 0.13526 & 0.06039 \\ 0.07799 & 0.26212 & 1.0000 & 0.51229 \\ 0.07494 & 0.17222 & 0.59962 & 1.0000 \end{bmatrix}$$

For example in the above matrix ρ_{12} & ρ_{34} are computed as,

$$\rho_{12} = \frac{8(0.05)^2(1+0.35)0.35^{1.5}}{(1-0.35^2)^2 + 4(0.05)^2(0.35)(1+0.35)^2} = 0.01294$$

$$\rho_{34} = \frac{8(0.05)^2(1+0.84)0.84^{1.5}}{(1-0.84^2)^2 + 4(0.05)^2(0.35)(1+0.84)^2} = 0.51229$$

Storey shear forces are computed by combining shear forces of different modes as follows,

$$V_4 = \sqrt{\{\lambda_4\}\left[\rho_{ij}\right]\{\lambda_4\}^T}$$

$$\lambda_4 = \{233.30 \quad -66.66 \quad 32.48 \quad -8.26\}$$

$$\{\lambda_4\}^T = \{233.30 \quad -66.66 \quad 32.48 \quad -8.26\}^T$$

$$\therefore V_4 = \sqrt{\{\lambda_4\}\begin{bmatrix} 1.0000 & 0.01294 & 0.00460 & 0.00311 \\ 0.09597 & 1.0000 & 0.13526 & 0.06039 \\ 0.07799 & 0.26212 & 1.0000 & 0.51229 \\ 0.07494 & 0.17222 & 0.59962 & 1.0000 \end{bmatrix}\{\lambda_4\}^T}$$

$$V_4 = \sqrt{57747}$$

$$V_4 = 240.31\ kN$$

Simillarly,

$$\mathrm{V}_3 = 532.85\ kN$$

$$\mathrm{V}_2 = 706.97\ kN$$

$$\mathrm{V}_1 = V_{Base} = 826.01\ kN$$

Now, storey lateral forces are computed from storey shear forces

$$Q_4 = V_4 = 240.31\ kN$$

$$Q_3 = V_3 - V_4 = 532.85 - 240.31 = 292.31\ kN$$

$$Q_2 = V_2 - V_3 = 706.97 - 532.85 = 174.12\ kN$$

$$Q_1 = V_1 - V_2 = 826.01 - 706.97 = 119.04\ kN$$

Table: Summary of results from different methods of analyses

Storey level	Equivalent static Method		CQC Method		SRSS Method	
	Shear force V_i (kN)	**Lateral force F_i (kN)**	**Shear force V_i (kN)**	**Lateral force F_i (kN)**	**Shear force V_i (kN)**	**Lateral force F_i (kN)**
4	**426.53**	**426.53**	**240.31**	**240.31**	**244.94**	**244.94**
3	**737.35**	**310.83**	**532.85**	**292.31**	**510.42**	**265.48**
2	**875.50**	**138.14**	**706.97**	**174.12**	**708.64**	**198.22**
1	**910.03**	**34.54**	**826.01**	**119.04**	**820.63**	**111.99**

The storey lateral forces and shear forces computed from equivalent static method and response spectrum method (dynamic analysis) are compared in the above table. In case of dynamic analysis the responses computed from both CQC and SRSS are tabulated for purpose of comparing these two methods of combining the individual modal response contributions. Comparison clearly indicates that both CQC and SRSS techniques consistently yield comparable shear and lateral force distribution with insignificantly small variation. It is important to note that in this particular example the modal frequencies are well separated. In case of such a situation, it sufficient to implement SRSS combination rule because CQC is a comparatively tedious when calculations are carried out manually.

Note: In the previous example, the building considered herein was analysed using equivalent static load method, wherein the fundamental period (T_a) of the structure is obtained using equation given under Clause 7.6 of IS-1893 (2002). When dynamic analysis is carried out either by the Time History Method or by the Response Spectrum Method, the design base shear computed from dynamic analysis (V_B) shall be compared with a base shear calculated using a fundamental period T_a ($\bar{V}_B$), where T_a is as per Clause 7.6. If base shear obtained from dynamic analysis (V_B) is less than base shear computed from equivalent static load method ($\bar{V}_B$ *i.e., using* T_a *as per Clause 7.6*), then as per Clause 7.8.2, all the response quantities (for example member forces, displacements, storey forces, storey shears and base reactions) shall be multiplied by ratio $\dfrac{\bar{V}_B}{V_B}$.

In the above example, $\bar{V}_B = 910.03\ kN$

Base shear as calculated by response spectrum method (SRSS) is, $V_B = 820.63\ kN$

$$\therefore\ \frac{\bar{V}_B}{V_B} = \frac{910.03}{820.63} = 1.109$$

Thus, the seismic forces obtained above by dynamic analysis should be scaled up as follows:

$$Q_4 = 244.94 \times 1.109 = 271.64\ kN$$
$$Q_3 = 265.48 \times 1.109 = 294.42\ kN$$
$$Q_2 = 198.22 \times 1.109 = 219.83\ kN$$
$$Q_1 = 111.99 \times 1.109 = 124.20\ kN$$

EXAMPLE: 3 (Problem from VTU Question Paper)

Subject code: 06CV834, Exam: December 2010

For a four storeyed RCC office building located in zone V and resting on hard rock, compute the seismic forces as per IS-1893-2002 equivalent static procedure. Height of first is 4.2 m and the remaining three stories are of height 3.2 m each. Plan dimensions (length and width) of the structure are 15 m x 20 m. The RCC frames are infilled with brick masonry.

Dead load on floor 12 kN/m^2 on floors and 10 kN/m^2 on roof. Live = 4 kN/m^2 on floors and 1.5 kN/m^2 on roof.

Also compute the base shear, neglecting the stiffness of infill walls. Compare the base shears for the two cases and comment on the result. **(20 Marks)**

Solution

Given data

Floor area = 15×20=300 m^2

Dead load: on floor = 12 kN/m^2

On roof = 10 kN/m^2

Live load: on floor = 4 kN/m^2

On roof = 1.5 kN/m^2

Note: Only 50% of the live load is lumped at the floors. At roof, no live load is to be lumped

Zone V, Z = 0.36

Assume SMRF thus, R = 5, Soil type = Hard Rock (Type-I)

Load at floor levels:

$$\text{Floors}: W1 = W2 = W3 = [12 + (0.5\times4)]\times300 = 4200 \text{ kN}$$
$$\text{Roof}: W4 = 10\times300 = 3000 \text{ kN}$$

Total seismic weight:

$$W = W1 + W2 + W3 + W4 = (3\times4200) + 3000 = \mathbf{15600\ kN}$$

Total height of the building:

$$h = (3.2\times3) + 4.2 = \mathbf{13.8\ m}$$

CASE – 1: With infill walls

Fundamental natural period :

(Moment resisting frame with in - fill walls)

a) Along 20 m direction :

$$T_a = 0.09\frac{h}{\sqrt{d}} = 0.09\frac{13.8}{\sqrt{20.0}} = 0.2777$$

b) Along 15 m direction :

$$T_a = 0.09\frac{h}{\sqrt{d}} = 0.09\frac{13.8}{\sqrt{15.0}} = 0.3207$$

Along 20 m direction

(Hard Rock),

For Ta=0.2777 s Sa/g = 2.5

For Zone V, Z = 0.36

Importance Factor, I = 1.0

Response Reduction Factor, R = 5.0 (SMRF)

Along 15 m direction

(Hard Rock),

For Ta=0.3207 s Sa/g = 2.5

For Zone V, Z = 0.36

Importance Factor, I = 1.0

Response Reduction Factor, R = 5.0 (SMRF)

Note: Since, Sa/g, Z, I and R values are same for both principal directions, it is sufficient calculate lateral forces in any one the principal axis.

Calculation of Base shear

a) A_h & V_B Along 20 m direction:

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(2.5)\left(\frac{1}{5}\right) = 0.09$$

$$V_B = A_h W = 0.09 \times 15600 = \mathbf{1404.00\ kN}$$

b) A_h & V_B Along 15 m direction:

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(2.5)\left(\frac{1}{5}\right) = 0.09$$

$$V_B = A_h W = 0.09 \times 15600 = \mathbf{1404.00\ kN}$$

Storey lateral forces and shear forces are calculated and tabulated in the following table.

Floor level (i)	W_i (kN)	h_i (m)	$W_i h_i^2$ (kN-m^2)	Storey forces $Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}$	Storey shear forces [V_i] (Cumulative sum) (kN)	
					Along 20 m Direction	Along 15 m Direction
4	3000	13.8	571320	595.36	595.36	595.36
3	4200	10.6	471912	491.77	1087.13	1087.13
2	4200	7.4	229992	239.67	1326.79	1326.79
1	4200	4.2	74088	77.21	**1404.00**	**1404.00**

CASE – 2: Without infill walls

Fundamental Natural period :
(Moment resisting frame without in-fill walls)
a) Along 20 m & 15 m directions:
$$T_a = 0.075h^{0.75} = 0.075 \times 13.8^{.75} = 0.537 \text{ sec}$$

Along 20 m & 15 m directions

(Hard Rock),

For Ta=0.537 s Sa/g = 1/T = 1/0.537= 1.862
For Zone V, Z = 0.36
Importance Factor, I = 1.0
Response Reduction Factor, R = 5.0 (SMRF)

Base shear:

a) A_h & V_B Along 20 m & 15 m directions:
$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(1.862)\left(\frac{1}{5}\right) = 0.06704$$
$$V_B = A_h W = 0.06704 \times 15600 = \mathbf{1045.81\ kN}$$

Storey lateral forces and shear forces are calculated and tabulated in the following table.

Floor level (*i*)	W_i (*kN*)	h_i (*m*)	$W_i h_i^2$ (*kN-m*^{2})	Storey forces $Q_i = V_B \dfrac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}$	Storey shear forces [V_i] (Cumulative sum) (kN	
					Along 20 m Direction	Along 15 m Direction
4	3000	13.8	571320	443.47	443.47	443.47
3	4200	10.6	471912	366.31	809.78	809.78
2	4200	7.4	229992	178.52	988.30	988.30
1	4200	4.2	74088	57.51	**1045.81**	**1045.81**

EXAMPLE: 4 (Problem from VTU Question Paper)

Subject code: 06CV834, Exam: June/July 2011

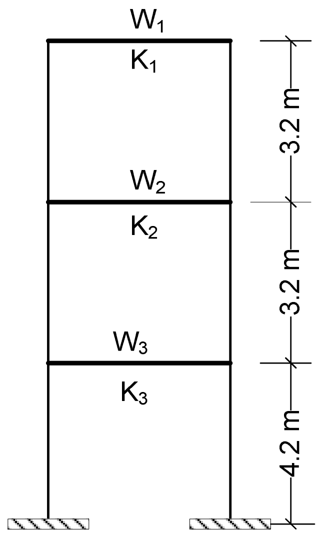
For the residential RCC (SMRF) building founded on soft soil and situated in zone V shown in figure. Compute the seismic forces for each storey using dynamic analysis procedure. Given the free vibration analysis results as follows,

Frequency: $\{\omega\} = \{47.832 \; 120.155 \; 167.00\}$ $rad/\sec$

Modes: $\{\phi_1\} = \begin{Bmatrix} 1.00 \\ 0.759 \\ 0.336 \end{Bmatrix}$ $\{\phi_2\} = \begin{Bmatrix} 1.00 \\ -0.805 \\ -1.157 \end{Bmatrix}$ $\{\phi_3\} = \begin{Bmatrix} 1.00 \\ -2.427 \\ 0.075 \end{Bmatrix}$

Seismic weights: $W_1 = W_2 = W_3 = 1962 \; kN$

Stiffness: $k_1 = k_2 = 160 \times 10^3 \; kN/m$ and $k_3 = 240 \times 10^3 \; kN/m$



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Solution

Given data

W3=W2=W1=1962 kN,

Frame: SMRF, R = 5,

Zone V: Z = 0.36

Soil type: Soft soil (Type-III)

Structure type: Residential, I=1

Free vibration characteristics:

Modes	Natural Period (sec)	Mode shapes		
		3rd Floor	2nd Floor	1st Floor
Mode 1	0.131	1.000	0.759	0.336
Mode 2	0.052	1.000	-0.805	-1.157
Mode 3	0.038	1.000	-2.427	0.075

Calculation of modal mass and modal participation factor (clause 7.8.4.5):

Storey Level	Seismic weight (W_i), kN	MODE-1		
		ϕ_{i1}	$W_i\ \phi_{i1}$	$W_i\ \phi_{i1}^{\ 2}$
3	1962.00	1.000	1962	1962.00
2	1962.00	0.759	1489.16	1130.27
1	1962.00	0.336	659.23	221.50
Σ	**5886.00**		4110.39	3313.77
Modal mass $M_1 = \dfrac{\left[\sum W_i\phi_{i1}\right]^2}{g\sum W_i\phi_{i1}^2}$		$= \dfrac{3313.77^2}{4110.39g} =$ 5098.512 kN/g		
% of Total weight		86.62 %		
Modal participation factor, $P_1 = \dfrac{\sum W_i\phi_{i1}}{\sum W_i\phi_{i1}^2}$		$= \dfrac{4110.39}{3313.77} =$ **1.2404**		

Storey Level	Seismic weight (W_i), kN	MODE-2		
		ϕ_{i2}	$W_i\ \phi_{i2}$	$W_i\ \phi_{i2}^2$
3	1962.00	1.000	1962.00	1962.00
2	1962.00	-.805	-1579.41	1130.27
1	1962.00	-1.157	-2270.03	221.50
Σ	**5886.00**		-1887.44	5859.85
Modal mass, $M_2 = \dfrac{\left[\sum W_i\phi_{i2}\right]^2}{g\sum W_i\phi_{i2}^2}$		$= \dfrac{-1887.44^2}{5859.85g} =$ 607.94 kN/g		
% of Total weight		10.34 %		
Modal participation factor, $P_2 = \dfrac{\sum W_i\phi_{i2}}{\sum W_i\phi_{i2}^2}$		$= \dfrac{-1887.44}{5859.85} =$ **-0.322**		

Storey Level	Seismic weight (W_i), kN	MODE-3		
		ϕ_{i3}	$W_i\ \phi_{i3}$	$W_i\ \phi_{i3}^2$
3	1962.00	1.000	1962	1962
2	1962.00	-2.247	-4761.77	11556.83
1	1962.00	0.075	147.15	11.03625
Σ	**5886.00**		-2652.62	13529.86
Modal mass, $M_3 = \dfrac{\left[\sum W_i\phi_{i3}\right]^2}{g\sum W_i\phi_{i3}^2}$		$= \dfrac{-2652.62^2}{13829.56g} =$ 520.066 kN/g		
% of Total weight		8.84 %		
Modal participation factor, $P_3 = \dfrac{\sum W_i\phi_{i3}}{\sum W_i\phi_{i3}^2}$		$= \dfrac{-2652.62}{13529.86} =$ **-0.196**		

MODE-1:

$$T_1 = 0.131\ s$$

$$\frac{S_a}{g} = 2.5....(0.10 \le T_1 \le 0.67 - \text{Soft soil})$$

$$A_{h(1)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(2.5)\frac{1}{5} = 0.09$$

$$Q_{i1} = A_{h(1)}\ \phi_{i1}\ P_1\ W_i = 0.09 \times 1.2404 \times (\phi_{i1}\ \ W_i) = 0.11164(\phi_{i1}\ \ W_i)$$

MODE-2:

$T_2 = 0.052\ s\ (T_2 \le 0.10)$

$\frac{S_a}{g} = 1 + (15 \times 0.052) = 1.78\ \ .....\ (0 \le T_2 \le 0.10,\ Soft\ soil)$

$A_{h(2)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(1.78)\frac{1}{5} = 0.0641$

$\therefore A_{h(2)} = \frac{Z}{2} = \frac{0.36}{2} = 0.18 > 0.0641$

$Q_{i2} = A_{h(2)}\ \phi_{i2}\ P_2\ W_i = 0.18 \times -0.322 \times (\phi_{i2} W_i) = -0.05796(\phi_{i2}\ W_i)$

MODE-3:

$T_3 = 0.038\ s\ (T_2 \le 0.10)$

$\frac{S_a}{g} = 1 + (15 \times 0.052) = 1.57\ \ .....\ (0 \le T_3 \le 0.10,\ Soft\ soil)$

$A_{h(3)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(1.57)\frac{1}{5} = 0.0565$

$\therefore A_{h(3)} = \frac{Z}{2} = \frac{0.36}{2} = 0.18 > 0.0565$

$Q_{i3} = A_{h(3)}\ \phi_{i3}\ P_3\ W_i = 0.18 \times -0.196 \times (\phi_{i3} W_i) = -0.03528(\phi_{i3}\ W_i)$

Lateral load calculation by modal analysis – SRSS method

Storey level	Weight *Wi* (kN)	**Mode – 1** [$Q_{i1} = 0.11164(\phi_{i1}\ W_i)$]		
		ϕ_{i1}	Q_{i1}	V_{i1}
3	1962.00	1.000	219.04	**219.04**
2	1962.00	0.759	166.25	**385.29**
1	1962.00	0.336	73.60	**458.88**

Storey level	Weight *Wi* (kN)	**Mode – 2** [$Q_{i2} = -0.05796(\phi_{i2}\ W_i)$]		
		ϕ_{i2}	Q_{i2}	V_{i2}
3	1962.00	1.000	-113.72	**-113.72**
2	1962.00	-.805	91.54	**-22.18**
1	1962.00	-1.157	131.57	**109.39**

Storey level	Weight *Wi* (kN)	**Mode – 3** [$Q_{i3} = -0.03528(\phi_{i3}\ W_i)$]		
		ϕ_{i3}	Q_{i3}	V_{i3}
3	1962.00	1.000	-69.22	**-69.22**
2	1962.00	-2.247	155.54	**86.32**
1	1962.00	0.075	-5.19	**81.13**

<u>SRSS method (Clause 7.8.4.4 – IS1893-2002)</u>:

The contribution of different modes are combined by Square Root of the Sum of the Squares (SRSS) using the following relationship, $V_i = \sqrt{V_{i1}^2 + V_{i2}^2 + V_{i3}^2}$

Then, storey lateral forces are calculated by, $F_i = V_i - V_{i+1}$.

The results obtained are tabulated in the following table.

Storey level	V_{i1}	V_{i2}	V_{i3}	**Combined shear force (SRSS) V_i (kN)**	**Combined lateral force (SRSS) F_i (kN)**
3	**219.04**	**117.25**	**69.22**	**256.32**	**256.32**
2	**385.29**	**22.86**	**-86.32**	**401.13**	**144.81**
1	**458.88**	**-112.79**	**-81.13**	**478.66**	**77.53**

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